Math 323
Linear Algebra and Matrix Theory I
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Key Homework 10

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Not satisfied are (1), (2) and (8).

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(a) c**x** is not in the set for negative values of c, so the set is not closed under scalar multiplication; and also the set contains no zero element.

(b) Observe that if we denote the new vector addition and scalar multiplication by $\oplus$ and $\otimes$ respectively, then if $\vec{x} = a$ and $\vec{y} = b$: $c \otimes (\vec{x} \oplus \vec{y}) = c \otimes (a\,b) = (a\,b)^c$ and $c \otimes \vec{x} \oplus c \otimes \vec{y} = a^c \oplus b^c = a^c b^c = (a\,b)^c$, where a and b denote positive real numbers; the number 1 is the zero vector $\vec{0}$ because $\vec{x} + \vec{0} = a\,1 = a$ for all $\vec{x}$ in the vector space ; to obtain the additive inverse of $\vec{2}$ the equation $\vec{2} \oplus \vec{y} = \vec{0} \Leftrightarrow 2\,b = 1$ needs to be solved for b, the result is $-\vec{2} = \vec{y} = \frac{1}{2}$.

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Usually the vector space of all, 2 by 2 matrices with the usual vector addition and scalar multiplication is denoted by $M_{2\,2}$; the zero vector in this space is $[\begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix}]$; $\frac{1}{2}A = [\begin{matrix} 1 & -1 \\ 1 & -1 \end{matrix}]$; $-A = [\begin{matrix} -2 & 2 \\ -2 & 2 \end{matrix}]$; and the smallest subspace W of $M_{2\,2}$ containing A, is the set of all multiples cA. Formally written: $W = \{cA \mid c \in R\}$.

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a) Example 1: $W = \{cA \mid c \in R\}$; Example 2: if $F = [\begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix}]$, $W = \{cA + dF \mid c, d \in R\}$

b) Yes, because $A - B = A + (-B) = I$.

c) Example: $W = \{cF \mid c \in R\}$, where F is defined as under (a).

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(a), (d), (e) are subspaces, they are all closed under vector addition and scalar multiplication. If W denotes the set under consideration, then in (b) W is neither closed under vector addition nor under scalar multiplication $(1,0,0)+(1,0,0)=(2,0,0)\notin W$ and $2\,(1,0,0)=(2,0,0)\notin W$; in (c) W is not closed under vector addition $(1,1,0)+(1,0,1)=(2,1,1)\notin W$; in (f) W is not closed under scalar multiplication $-2\,(1,2,3)=(-1,-2,-3)\notin W$.