Math 323
Linear Algebra and Matrix Theory I
Fall 1999

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Key Homework 11

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a) $W = \{ c \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + d \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \mid c, d \in R \}$.

b) $W = \{ c \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \mid c \in R \}$.

c) $W = \{ c \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + d \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mid c, d \in R \} = \{ c \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + d \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \mid c, d \in R \}$, which is the set of all diagonal matrices.

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a) The subspaces of R^2 are R^2 itself, lines $\vec{n} \bullet \vec{x} = 0$; and $\{(0,0)\}$.

b) The subspaces of R^4 are R^4 itself, three dimensional "planes" through the origin $\vec{n} \bullet \vec{x} = 0$; two dimensional planes through the origin $\vec{n}_1 \bullet \vec{x} = 0$ and $\vec{n}_2 \bullet \vec{x} = 0$; one dimensional "planes" (lines) through the origin $\vec{n}_1 \bullet \vec{x} = 0$, $\vec{n}_2 \bullet \vec{x} = 0$ and $\vec{n}_3 \bullet \vec{x} = 0$,and finally $Z = \{(0,0,0,0)\}$. Observe that the three, two and one dimensional planes can alternatively be described parametrically by $\{c\,\vec{v}_1 + d\,\vec{v}_2 + e\,\vec{v}_3 \mid c, d, e \in R\}$, $\{c\,\vec{v}_1 + d\,\vec{v}_2 \mid c, d \in R\}$, and $\{c\,\vec{v} \mid c \in R\}$;

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a) The set of invertible , 2 by 2, matrices is, for instance, not closed under scalar multiplication $0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. (it is not closed under vector addition either, can you give an example?)
A even quicker argument is this: "the set does not contain the zero matrix, therefore it cannot be a vectorspace", and, of course, every subspace of a vectorspace is a vectorspace.

b) The set of singular, 2 by 2 matrices is closed under scalar multiplication, but not closed under vector addition $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$.

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a) There is a solution only if $\vec{b}$ is in the column space of $A \Rightarrow b_2 = 2b_1, b_3 = -b_1$.

b) There is a solution only if $\vec{b}$ is in the column space of $A \Rightarrow b_3 = -b_1$.

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… unless $\vec{b}$ is a linear combination of the columns of A, that means unless $\vec{b}$ is in the column space of A. An example where the column space gets larger is given by $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

An example where the column space does not get larger is given by $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$.

The system $A\vec{x} = \vec{b}$ is solvable if and only if $\vec{b}$ is in the columnspace of A, and that happens exactly when the column spaces are the same for A and $[A\ \vec{b}\,]$.