**Math 323**
**Linear Algebra and Matrix Theory I**
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# Key Homework 12

## Strang Page 118 no: 1

a)

```
A=[1 2 2 4 6; 1 2 3 6 9; 0 0 1 2 3]

A =
     1     2     2     4     6
     1     2     3     6     9
     0     0     1     2     3

A1=rowcomb(A, 1, 2, -1)

A1 =
     1     2     2     4     6
     0     0     1     2     3
     0     0     1     2     3

A2=rowcomb(A1, 2, 3, -1)

A2 =
     1     2     2     4     6
     0     0     1     2     3
     0     0     0     0     0
```

Clearly $x_2, x_4$, and $x_5$ are the free variables, $x_1$ and $x_3$ are the pivot variables.

b)

```
B=[2 4 2; 0 4 4; 0 8 8]

B =
     2     4     2
     0     4     4
     0     8     8

B1=rowcomb(B, 2, 3, -2)

B1 =
     2     4     2
     0     4     4
     0     0     0
```

$x_3$ is the free variable, $x_1$ and $x_2$ are the pivot variables.

## Strang Page 118 no: 2

a)
Let $x_2 = 1$ and the other free variables equal to zero, then $x = (-2,1,0,0,0)$.
Let $x_4 = 1$ and the other free variables equal to zero, then $x = (0,0,-2,1,0)$.
Let $x_5 = 1$ and the other free variables equal to zero, then $x = (0,0,-3,0,1)$.

This can be verified using the nulbasis command.

```
nulbasis(A)

ans =
    -2     0     0
     1     0     0
     0    -2    -3
     0     1     0
     0     0     1
```

b)
Let $x_3 = 1$ and the other free variables equal to zero, then $x = (1,-1,1)$.

Again the nulbasis command will readily verify this result.

```
nulbasis(B)

ans =
     1
    -1
     1
```

## Strang Page 118 no: 5

a)

```
A=[-1  3 5; -2 6 10]

A =
    -1     3     5
    -2     6    10

A2=rowcomb(A,1, 2, -2), E21=rowcomb(eye(2),1, 2, -2)

A2 =
    -1     3     5
     0     0     0
E21 =
     1     0
    -2     1
```

```
L=inv(E21), U=A2, LU=L*U, A

L =
     1     0
     2     1
U =
    -1     3     5
     0     0     0
LU =
    -1     3     5
    -2     6    10
A =
    -1     3     5
    -2     6    10
```

b)

```
A=[-1  3 5; -2 6 7]

A =
    -1     3     5
    -2     6     7

A2=rowcomb(A,1, 2, -2), E21=rowcomb(eye(2),1, 2, -2)

A2 =
    -1     3     5
     0     0    -3
E21 =
     1     0
    -2     1

L=inv(E21), U=A2, LU=L*U, A

L =
     1     0
     2     1
U =
    -1     3     5
     0     0    -3
LU =
    -1     3     5
    -2     6     7
A =
    -1     3     5
    -2     6     7
```

## **Strang Page 118 no: 6**

The special solutions can be deduced from the U matrices:

a) $\vec{s}_1 = (3,1,0), \vec{s}_2 = (5,0,1)$
b) $\vec{s} = (3,1,0)$

… the number of free variables plus the number of pivot variables equals n.

## Strang Page 118 no: 7

a) Read the equation of the plane from the U matrix: $-x+3y+5z=0$. The vector representation is given by: $\vec{x}=\alpha\,\vec{s}_1+\beta\,\vec{s}_2$.

b) Read the equations of the line from the U matrix: $-x+3y+5z=0,-3z=0$. Recall that a line in 3-space is represented by two linear equations, it is the intersection of those two planes. The vector representation is given by: $\vec{x}=\alpha\,\vec{s}$.

## Strang Page 118 no: 8

a)

```
A=[-1 3 5; -2 6 10], R=rref(A)

A =
    -1     3     5
    -2     6    10
R =
     1    -3    -5
     0     0     0

I=R(1, 1)

I =
     1
```

b)

```
A=[-1 3 5; -2 6 7], R=rref(A)

A =
    -1     3     5
    -2     6     7
R =
     1    -3     0
     0     0     1

I=[R(:, 1) R(:, 3)]

I =
     1     0
     0     1
```

## Strang Page 118 no: 9

a) False! Look at the matrix A defined by:

```
v=[1 2 3]'; A=[[2 1 7]' v 2*v], rref(A)

A =
     2     1     2
     1     2     4
     7     3     6
ans =
     1     0     0
     0     1     2
     0     0     0
```

b) True! If A is invertible, then rref(A) = I.

c) True! Because the sum of the number of pivot variables and the number of free variables equals n.

d) True! Because there can be only one pivot in each row.

## Strang Page 118 no: 12

a)

```
1 x 0 x x x 0 0
0 0 1 x x x 0 0
0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 1
```

b)

```
0 1 x 0 0 x x x
0 0 0 1 0 x x x
0 0 0 0 1 x x x
0 0 0 0 0 0 0 0
```

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… when the matrix has 5 pivots. The column space is $R^5$ when there are 5 pivots, because rref(A) = I and $A\vec{x}=\vec{b}$ is solvable $\forall\ \vec{b}\in R^5$.

## Strang Page 118 no: 18

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} + x\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

## Strang Page 118 no: 21

a)
```
A=[1 0 -2 -3; 0 1 -2 -1]

A =
     1     0    -2    -3
     0     1    -2    -1
```

## Strang Page 118 no: 24

This is not possible. Observe that both the alleged nullspace and the alleged column space are subspaces of $R^3$. This means that the matrix A must be, 3 by 3. Having (0, 0, 1) in the nullspace, means that the third column of $A$ equals the zero vector. Having (1, 0, 1) in the null space means that the first and third column of $A$ are equal, so both columns equal the zero vector. That leaves just the second column to generate the vectors (1,1,0) and (0,1,1), which is not possible.

## Strang Page 118 no: 27

The column space is the set of all linear combinations of the pivot columns (it will be proved later that we cannot do with less), the null space is the set of all linear combinations of the "special" solutions to $A\vec{x} = \vec{0}$. If there are $r$ of these special solutions, there must be $3-r$ pivot columns. the equation $r = 3-r$ would imply that the number 3 is even, which is impossible.

## Strang Page 118 no: 32

If the nullspace of $A$ consists of all multiples of $\vec{x} = (2,1,0,1)$, then there is only one free variable, there must the be three pivot variables, so $U$ must have three pivots.