**Math 323**
**Linear Algebra and Matrix Theory I**
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# Key Homework 15

## Strang Page 151 no: 1

```
A=[1 1 1 2; 0 1 1 3; 0 0 1 4]

A =
     1     1     1     2
     0     1     1     3
     0     0     1     4

R=rref(A)

R =
     1     0     0    -1
     0     1     0    -1
     0     0     1     4
```

Clearly the first three vectors are linearly independent, while the fouth vector is a linear combination of the first three.

## Strang Page 151 no: 2

We find the dimension of the column space, which equals the rank of A.

```
A=[1 1 1 0 0 0; -1 0 0 1 1 0; 0 -1 0 -1 0 1; 0 0 -1 0 -1 -1]

A =
     1     1     1     0     0     0
    -1     0     0     1     1     0
     0    -1     0    -1     0     1
     0     0    -1     0    -1    -1

R=rref(A)

R =
     1     0     0    -1    -1     0
     0     1     0     1     0    -1
     0     0     1     0     1     1
     0     0     0     0     0     0
```

There are three pivot columns, so the dimension of the column space equals 3. This is easily verified using the MATLAB's **rank** command.

```
rank(A)

ans =
     3
```

## Strang Page 151 no: 4

If a, d and f are all non-zero, then they are all pivots. This means that the reduced row echelon form of U is the identity matrix, which implies that U is invertible, which implies that Ux = 0 has only the trivial solution, and therefore the columns of U are linearly independent.

## Strang Page 151 no: 5

a)

```
rref([1 2 3; 3 1 2; 2 3 1])

ans =
     1     0     0
     0     1     0
     0     0     1
```

The vectors are linearly independent.

b)

```
rref([1 2 -3; -3 1 2; 2 -3 1])

ans =
     1     0    -1
     0     1    -1
     0     0     0
```

These vectors are linearly dependent.

## Strang Page 151 no: 7

Let $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0} \Rightarrow c_1(\overline{w}_2 - \overline{w}_3) + c_2(\overline{w}_1 - \overline{w}_3) + c_3(\overline{w}_1 - \overline{w}_2) = \vec{0}$

$\Rightarrow (c_2 + c_3)\overline{w}_1 + (c_1 - c_3)\overline{w}_2 + (c_1 + c_2)\overline{w}_3 = \vec{0}$

$\Rightarrow (c_2 + c_3) = 0, (c_1 - c_3) = 0, \text{ and } (c_1 + c_2) = 0$, because the w vectors are linearly independent. We use MATLAB to show that this system has a non-trivial solution.

```
A=[0 1 1; 1 0 -1; 1 1 0], R=rref(A)

A =
     0     1     1
     1     0    -1
     1     1     0
R =
     1     0    -1
     0     1     1
     0     0     0
```

Since the R matrix has a free column there are nontrivial solutions. One such solution is produced by the nulbasis command.

```
dependency=nulbasis(A)

dependency =
     1
    -1
     1
```

This shows that $\vec{v}_3$ equals $\vec{v}_2$ minus $\vec{v}_1$.

## Strang Page 151 no: 9

a) … because the maximum number of linearly independent vectors in $R^3$ equals 3.
b) … if they are scalar multiples of each other.
c) … because $0\,\vec{v}_1 = \vec{0}$.

## Strang Page 151 no: 11

a) It's a line in $R^3$.
b) It's a plane in $R^3$.
c) It's a plane in $R^3$.
d) It's all of $R^3$, because there exist three linearly independent vectors with positive components, and since the dimension of $R^3$ equals 3, these vectors must span $R^3$.

## Strang Page 151 no: 12

a) … $A\vec{x} = \vec{b}$.
b) … $A^T\vec{y} = \vec{c}$
c) False, the zero vector is part of any vector space, so the zero vector is always in the row space.

## Strang Page 151 no: 13

```
A=[1 1 0; 1 3 1; 3 1 -1], [L, U]=slu(A)

A =
     1     1     0
     1     3     1
     3     1    -1
Small pivot encountered in column 3.
L =
     1     0     0
     1     1     0
     3    -1     1
U =
     1     1     0
     0     2     1
     0     0     0
```

The dimension of column and row spaces of A and the column and row spaces of U all equal the rank of A, which equals the rank of U and clearly is 2. The row spaces of A and U are the same, the column spaces of A and U are different.

## Strang Page 151 no: 16

… has dimension n
… are a basis
… $m \geq n$

## Strang Page 151 no: 17

a) B={(1, 1, 1, 1)}
b) B={(1, 0, 0, -1), (0, 1, 0, -1), (0, 0, 1, -1)}
c)

```
B=nulbasis([1 1 0 0; 1 0 1 1])

B =
    -1    -1
     1     1
     1     0
     0     1
```

d) A basis for the column space is B={(1, 0), (0, 1)} and a basis for the null space is

```
B=nulbasis([1 0 1 0 1; 0 1 0 1 0])

B =
    -1     0    -1
     0    -1     0
     1     0     0
     0     1     0
     0     0     1
```

## ATLAST Page 67 no: 1

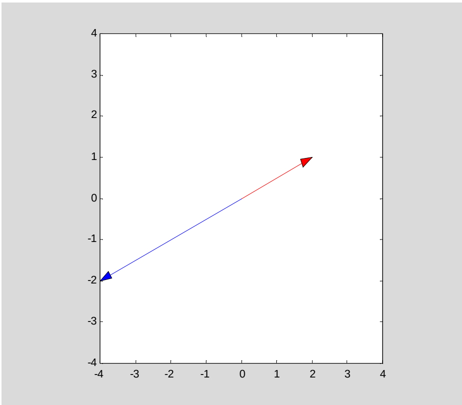
a) Two vectors are linearly dependent if and only if one is a scalar multiple of the other. That is if the vectors are parallel.

First we graph two linearly dependent vectors

```
v1=[2; 1], v2=-2*v1

v1 =
     2
     1
v2 =
    -4
    -2
```

```
drawvec(v1, 'red', 4); hold on, drawvec(v2, 'blue', 4);
```

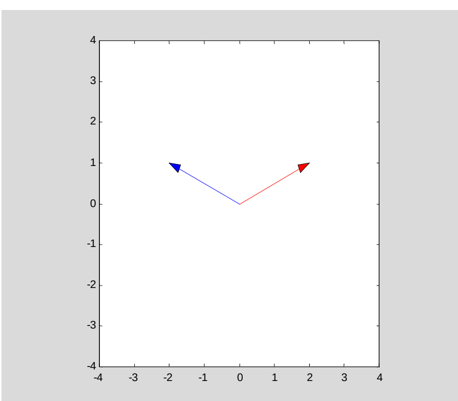


Then we graph two linearly independent vectors

```
v1=[2; 1], v2=[-2; 1]
```

```
v1 =
     2
     1
v2 =
    -2
     1
```

```
drawvec(v1, 'red', 4); hold on, drawvec(v2, 'blue', 4);
```



b) $\vec{v}_3$ is a linear combination of $\vec{v}_1$ and $\vec{v}_2$ if $\vec{v}_3$ is in the plane spanned by $\vec{v}_1$ and $\vec{v}_2$.

c) Three vectors in $R^3$ are linearly dependent if they lie in the same plane.

## ATLAST Page 67 no: 2

a) The span of $\vec{v}_1$ is a line through the origin parallel to $\vec{v}_1$.

b) Then the span of $\{\vec{v}_1, \vec{v}_2\}$ is a plane through the origin.

## ATLAST Page 67 no: 5

a)

i) Compute the rank of V. If it equals k, then the vectors are linearly independent.

ii) Compute the rank of V. If it equals n, then the vectors span the $R^n$.

iii) Use the **partic** command to find the coefficients of a possible linear combination.

iv) Apply the rowbasis command to V.

b)

```
V=[2 3 3; -3 4 1; 0 1 3; -1 2 -3], w1=[1 0 0 1]', w2=[1 1 0 1]'

V =
      2             3             3
     -3             4             1
      0             1             3
     -1             2            -3
w1 =
      1
      0
      0
      1
w2 =
      1
      1
      0
      1
```

i)

```
rank(V)

ans =
      3
```

The vectors are linearly independent.

ii) The vectors do not span the $R^4$.

iii)

```
coeff1=partic(V, w1), coeff2=partic(V, w2)

coeff1 =
     []
coeff2 =
     1/8
     3/8
    -1/8
```

Clearly w1 is not a linear combination of the columns of V, w2 is a linear combination of those columns. $\vec{w}_2 = \frac{1}{8}\vec{v}_1 + \frac{3}{8}\vec{v}_2 - \frac{1}{8}\vec{v}_3$.

iv) The dimension of the span of the columns of V is the dimension of the column space of V, which equals the rank of V, which equals 3. A basis for this span is given by the column vectors of V.

c)

```
V=[4 2 3 2 1; 3 -4 -3 2 0; -1 -4 1 3 -1; 3 -3 4 -2 1], w1=[1 0 0 1]',
w2=[1 1 0 1]'

V =
     4           2           3           2           1
     3          -4          -3           2           0
    -1          -4           1           3          -1
     3          -3           4          -2           1
w1 =
     1
     0
     0
     1
w2 =
     1
     1
     0
     1
```

i)

```
rank(V)

ans =
     4
```

The vectors are linearly dependent.

ii) The vectors do span the $R^4$.

iii) Since the column vectors of V span the $R^4$, both w1 and w2 must be linear combinations of these column vectors. Possible coefficients can be found as in part (b) of this problem.

```
coeff1=partic(V, w1), coeff2=partic(V, w2)

coeff1 =
    1/7
     0
    1/7
     0
     0
coeff2 =
   76/287
   -7/123
    4/313
    1/123
     0
```

iv) The span of the columns of V equals $R^4$. The dimension of this space is obviously 4, and a basis for it is given by the columns of the 4 by 4 identity matrix.