Math 323
Linear Algebra and Matrix Theory I
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Key Homework 16

Strang Page 152 no: 18

Since the column sapce of U is R^2, we can pick any basis for R^2. B1= {(1, 0), (0, 1)}, B2={(1, 0), (1, 1)}, B3={(4, 5), (-3, 7)}. As two different basis of the row space, we can take: B1:={(1, 0, 1, 0, 1), (0, 1, 0, 1, 0)}, B2={(1, 0, 1, 0, 1), (1, 1, 1, 1, 1)}.

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a) … might not span R^4.
b) … are not linearly independent.
c) … might be a basis for R^4.

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If they are linearly independent then rank(A) = n. If they span R^m, then rank(A) = m. If they are a basis for R^m, then not only does the rank of A again equal m, but also m = n.

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Let $y=\alpha_1, z=\alpha_2 \Rightarrow x=2\alpha_1-3\alpha_2 \Rightarrow \vec{x}=\alpha_1(2,1,0)+\alpha_2(-3,0,1)$. This implies that B={(2, 1,0), (-3, 0, 1)} forms a basis for this plane.

The intersection with the xy plane is given by the two equations z = 0 and x - 2y = 0.
Let $y=\alpha \Rightarrow x=2\alpha \Rightarrow \vec{x}=\alpha(2,1,0)$. So B={(2, 1, 0)} is a basis for the intersection of the two planes.

The left hand side of x - 2y + 3z =0 can be interpreted as the dot product of the vectors (1, -2, 3) and (x, y, z). Therefore a basis for all vectors perpendicular to the plane is given by B={(1, -2, 3)}

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a) True ! This can be done by a procedure established in class.
b) False ! Because one or more of the basis vectors might not be in S.

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A basis for the column space of A is {(1, 0, 1), (3, 1,3)}.
A basis for the column space of U is {(1, 0, 0), (3, 1,0)}.
The row spaces of A and U are equal. A basis is given by {(1, 3, 2), (0, 1, 1)}
The null spaces of A and U are equal. A basis is given by {(1, -1, 1)}

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a) A basis is given by:

```
E1=[1 0 0; 0 0 0; 0 0 0], E2=[0 0 0; 0 1 0; 0 0 0], E3=[0 0 0; 0 0 0; 0
0 1]
```

```
E1 =
     1     0     0
     0     0     0
     0     0     0
E2 =
     0     0     0
     0     1     0
     0     0     0
E3 =
     0     0     0
     0     0     0
     0     0     1
```

b) A basis is given by E1, E2 and E3 together with

```
E4=[0 1 0; 1 0 0 ; 0 0 0], E5=[0 0 1; 0 0 0; 1 0 0], E6=[0 0 0; 0 0 1; 0
1 0]
```

```
E4 =
     0     1     0
     1     0     0
     0     0     0
E5 =
     0     0     1
     0     0     0
     1     0     0
E6 =
     0     0     0
     0     0     1
     0     1     0
```

c) A basis is given by:

```
E7=[0 1 0; -1 0 0 ; 0 0 0], E8=[0 0 1; 0 0 0; -1 0 0], E8=[0 0 0; 0 0 1;
0 -1 0]
```

```
E7 =
     0     1     0
    -1     0     0
     0     0     0
```

```
E8 =
     0     0     1
     0     0     0
    -1     0     0
E8 =
     0     0     0
     0     0     1
     0    -1     0
```

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a) Create the coefficient matrix of the nine equations resulting from setting a linear combination of these six permutation matrices (page 92) equal to zero.

```
A=[1 0 0 0 1 0; 0 1 1 0 0 0; 0 0 0 1 0 1; 0 1 0 0 0 1; 1 0 0 1 0 0; 0 0
1 0 1 0; 0 0 1 1 0 0; 0 0 0 0 1 1; 1 1 0 0 0 0]
```

```
A =
     1     0     0     0     1     0
     0     1     1     0     0     0
     0     0     0     1     0     1
     0     1     0     0     0     1
     1     0     0     1     0     0
     0     0     1     0     1     0
     0     0     1     1     0     0
     0     0     0     0     1     1
     1     1     0     0     0     0
```

The coefficients of a non-trivial linear combination which equals zero can easily be found using the nulbasis command

```
nulbasis(A)
```

```
ans =
     1
    -1
     1
    -1
    -1
     1
```

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If $y(0) = 0$ then

$$A + B + C = 0 \Rightarrow C = -A - B \Rightarrow y(x) = A\cos(x) + B\cos(2x) + (-A - B)\cos(3x)$$
$$= A(\cos(x) - \cos(3x)) + B(\cos(2x) - \cos(3x))$$

This implies that the functions g(x) = cos(x) - cos(3x) and h(x) = cos(2x)-cos(3x) span the indicated vector space. To show that they are linearly independent we set the linear combination A g(x) + B h(x) equal to zero (for all x). If we now substitute x =1 and x = 2, and investigate the nullspace of the resulting coefficient matrix.

```
M=[cos(1)-cos(3) cos(2)-cos(3); cos(2)-cos(6) cos(4)-cos(6)],
nulbasis(M)
```

```
M =
    1.5303    0.5738
   -1.3763   -1.6138
ans =
   Empty matrix: 2-by-0
```

Clearly the only linear combination which gives zero is the trivial linear combination. This means that the functions g and h are linear independent, we already know they span the vector space, so they must be a basis for the vector space.

Be careful, singularity of the matrix M is not a sufficient condition for linear dependence of g and h. (since linear depence requires that a non trivial linear combination equals zero **for all x**)

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A basis for P_3 is given by $B = \{1, x, x^2, x^3\}$. Let $V = \{p \in P_3 | p(1) = 0\}$ If $p \in V$ and $p(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$, then $c_0 + c_1 + c_2 + c_3 = 0 \Rightarrow c_3 = -c_0 - c_1 - c_2$ $\Rightarrow p(x) = c_0(1 - x^3) + c_1(x - x^3) + c_2(x^2 - x^3)$. Clearly the space V is spanned by $f(x) = 1 - x^3, g(x) = x - x^3$ and $h(x) = x^2 - x^3$. Linear independence is proved in a way similar to that used in problem 34. Use x = 0, 2, 3 to generate the equations. You should stay away from x = 1 (do you know why?)

```
M=[1 0 0; -7 -6 -4; -26 -24 -18], nulbasis(M)
```

```
M =
     1     0     0
    -7    -6    -4
   -26   -24   -18
ans =
   Empty matrix: 3-by-0
```

We quickly conclude that {f(x), g(x), h(x)} forms a basis for V.

ATLAST Page 69 no: 6

a) The easiest way to find the row echelon form U of this matrix is to apply PA = LU decomposition to the matrix A.

```
format rat
A=[1 2 5 -2 1; -1 2 3 2 3; 4 -1 2 -1 2; 2 -1 0 3 4], [P, L, U]=plu(A); U
```

```
A =
     1     2     5    -2     1
    -1     2     3     2     3
     4    -1     2    -1     2
     2    -1     0     3     4
Pivots in columns:
     1     2     4
```

```
No pivots in columns:
     3     5
Pivots in rows:
     1     2     3
U =
     1     2     5    -2     1
     0     4     8     0     4
     0     0     0     7     7
     0     0     0     0     0
```

b) The rows of U form a basis for the row space of A.

c) Since $A = P^{-1}LU$ the coefficients of the desired linear combinations can be read from the matrix $P^{-1}L$.

```
PinvL=inv(P)*L

PinvL =
    1.0000         0         0         0
   -1.0000    1.0000         0         0
    4.0000   -2.2500    1.0000         0
    2.0000   -1.2500    1.0000    1.0000
```

For instance (row 4 of A) = 2 (row 1 of U) - 1.25 (row 2 of U) + (row 3 of U).
Observe that the fourth row of U does not play a role because it is a zero row!

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a)

```
rowbasis(A)

ans =
     1    -1     4
     2     2    -1
     5     3     2
    -2     2    -1
     1     3     2
```

b)

```
colbasis(A)

ans =
     1     2    -2
    -1     2     2
     4    -1    -1
     2    -1     3
```

c)

```
nulbasis(A)

ans =
    -1    -1
    -2    -1
     1     0
     0    -1
     0     1
```

d)

```
nulbasis(A')

ans =
     1
    -1
    -1
     1
```

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```
M=randint(5, 7, 5, 3), nullity=rank(nulbasis(M))

M =
     2     0     0    -4    -2    -1     2
     1    -1    -1     0    -1     1    -1
     0    -4    -4    -1     0    -3     1
     3     1     1    -3    -3     2     0
    -2     2     2     0     2    -2     2
nullity =
     4

M=randint(3, 4, 5, 3), nullity=rank(nulbasis(M))

M =
     1    -4     1    -1
     2    -1    -2     2
    -2     1     1    -1
nullity =
     1

M=randint(13, 9, 5, 5), nullity=rank(nulbasis(M))

M =
    -1     0     1    -1     0     4     0     0     1
     2     0    -1    -1     1    -1    -2     0    -1
     4    -4    -1     0     1     1    -1     1    -3
    -3     2     1     1    -1    -1     2     0     2
    -1     1     1    -2     2     4    -1     2     2
     2    -1    -1     0    -1    -1    -1    -2    -2
     0     2     1     0    -1    -1     1     1     1
     4    -2    -1     0    -1    -1    -1    -1    -3
     0    -1    -1     2     0    -4     1     0    -1
     0    -1    -1     0     3    -1    -1     2     0
    -2     0     1     2    -1    -1     3     1     1
     2    -2     0    -1     1     3    -1     1    -1
    -1     0     1     1     0     0     2     2     1
```

```
nullity =
     4
```

Observe that in each case the nullity equals n - r, a fact that will be fully substantiated in section 3.6 of the Strang textbook.