Math 323
Linear Algebra and Matrix Theory I
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Key Homework 17

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a) Use the procedure we introduced in class.

```
V=[5 -1; -7 2; 1 -1; 4 -4], R=rref([V eye(4)])

V =
     5    -1
    -7     2
     1    -1
     4    -4
R =
    1.0000         0         0   -0.2000         0   -0.1000
         0    1.0000         0   -0.2000         0   -0.3500
         0         0    1.0000    0.8000         0    0.1500
         0         0         0         0    1.0000   -0.2500
```

A basis for R^4 is given by the columns of the matrix B below.

```
H=[V eye(4)]; B=H(:, [1 2 3 5])

B =
     5    -1     1     0
    -7     2     0     0
     1    -1     0     1
     4    -4     0     0
```

b) Since the vectors $\vec{v}_1, \cdots, \vec{v}_k$ are linearly independent, none of them can be written as a linear combination of its predecessors. This implies that the first k columns of the matrix obtained by augmenting the V matrix with the identity matrix, will always be pivot columns.

c)

```
hopefully_B=[V randint(4, 2, 5, 2)], check=rank(hopefully_B)

hopefully_B =
     5    -1     4     3
    -7     2     1     0
     1    -1     0    -3
     4    -4     1     3
check =
     4
```

Because the rank of this matrix equals 4, its columns a linearly independent and form a basis for R^4.

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a) Use the method explained in class.

```
V=[3 4 -6 4; -4 0 0 8; -3 -2 3 1; 2 6 -9 11], R=rref(V)

V =
     3     4    -6     4
    -4     0     0     8
    -3    -2     3     1
     2     6    -9    11
R =
    1.0000         0         0   -2.0000
         0    1.0000   -1.5000    2.5000
         0         0         0         0
         0         0         0         0
```

The first two columns are pivot columns, and therefore the first two columns of V form a basis for C(V).

b) Use all sets of two linearly independent columns of V. (observe that the second and the third column are linearly dependent) The desired bases are: $\{\vec{v}_1,\vec{v}_2\},\{\vec{v}_1,\vec{v}_3\},\{\vec{v}_1,\vec{v}_4\},\{\vec{v}_2,\vec{v}_4\},\{\vec{v}_3,\vec{v}_4\}$.

c) Take the answer of (b) plus: $\{\vec{v}_1,\vec{v}_2,\vec{v}_3\},\{\vec{v}_1,\vec{v}_2,\vec{v}_4\},\{\vec{v}_1,\vec{v}_3,\vec{v}_4\},\{\vec{v}_2,\vec{v}_3,\vec{v}_4\},\{\vec{v}_1,\vec{v}_2,\vec{v}_3,\vec{v}_4\}$.

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a)

```
V=[3 -3 4; 4 1 1; 1 -4 2; 3 1 3], W=[-5 -1 2; 2 8 -11; -3 -4 5; -3 4 -4]

V =
     3    -3     4
     4     1     1
     1    -4     2
     3     1     3
W =
    -5    -1     2
     2     8   -11
    -3    -4     5
    -3     4    -4

X=W\V, Y=V\W

X =
   -1.0000    1.0000   -1.0000
   -2.0000    4.0000   -1.0000
   -2.0000    3.0000   -1.0000
Y =
    1.0000    2.0000   -3.0000
   -0.0000    1.0000   -1.0000
   -2.0000   -1.0000    2.0000
```

Let Sv and Sw denote the span of the v vectors and the w vectors respectively. Since V = WX , Sv is a subset of Sw. Similarly because W=VY, Sw is a subset of Sv. We conclude that Sv = Sw.

b)

```
X*Y, Y*X

ans =
    1.0000    0.0000   -0.0000
         0    1.0000   -0.0000
         0         0    1.0000
ans =
    1.0000    0.0000   -0.0000
         0    1.0000         0
    0.0000   -0.0000    1.0000
```

Since XY = I and YX = I, X and Y are each others inverse.

The reason for that is simple: V = WX and W = VY implies that V = VYX. This last equation expresses the columns of V as linear combinations of themselves, but since the columns of V are linearly independent, such linear combinations are unique and must therefore be the trivial linear combinations $\vec{v}_1 = \vec{v}_1, \cdots, \vec{v}_k = \vec{v}_k$, which means that XY = I.

c) Since the set $w = \{\vec{w}_1, \cdots, \vec{w}_k\}$ is linearly independent, span(w) has dimension k. Moreover, since V=WX, the linearly indepent vectors $\vec{v}_1, \cdots, \vec{v}_k$ all belong to span(w). This implies that $v = \{\vec{v}_1, \cdots, \vec{v}_k\}$ is a set of k linearly independent vectors in a vector space, span(w), of dimension k, which in turn means that v is a basis for span(w) and consequently span(v) = span(w).