Math 323
Linear Algebra and Matrix Theory I
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Key Homework 19

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a) False !

```
A=-eye(2), detA_plus_I=det(A+eye(2)), detA_plus_detI=det(A)+det(eye(2))

A =
    -1     0
     0    -1
detA_plus_I =
     0
detA_plus_detI =
     2
```

b) True ! $\det(ABC) = \det(A(BC)) = \det(A)\det(BC) = \det(A)\det(B)\det(C)$.

c) True ! $\det(A^4) = \det(A^3 A) = \det(A^3)\det(A) = \cdots = \det(A)^4$

d) False !

```
A=eye(2), det_4A=det(4*A), Four_detA=4*det(A)

A =
     1     0
     0     1
det_4A =
    16
Four_detA =
     4
```

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a) $\det(Q_1) = \cos^2(\theta) + \sin^2(\theta) = 1$

b) $\det(Q_2) = 1 - 2\sin^2(\theta) - 2\cos^2(\theta) = -1$

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In this computation the entire, 2 by 2, matrix is multiplied by $\dfrac{1}{ad-bc}$, this means that the determinant is multiplied by $\dfrac{1}{(ad-bc)^2}$, so the determinant of the inverse matrix equals:

$$\det(A^{-1}) = \frac{1}{(ad-bc)^2}(ad-bc) = \frac{1}{ad-bc} = \frac{1}{\det(A)}$$. (and that is exactly what we expect !)

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a)

```
A=[1 2 3 0; 2 6 6 1; -1 0 0 3; 0 2 0 5], [P, L, U]=plu(A);, U

A =
     1     2     3     0
     2     6     6     1
    -1     0     0     3
     0     2     0     5
Pivots in columns:
     1     2     3     4
Pivots in rows:
     1     2     3     4
U =
     1     2     3     0
     0     2     0     1
     0     0     3     2
     0     0     0     4
```

Since there are no row interchanges $\det(A) = \det(U)$ which equals the product of its diagonal elements.

```
det_A=prod(diag(U))

det_A =
    24
```

b)

```
A=[2 -1 0 0; -1 2 -1 0; 0 -1 2 -1; 0 0 -1 2], [P, L, U]=plu(A); U

A =
     2    -1     0     0
    -1     2    -1     0
     0    -1     2    -1
     0     0    -1     2
Pivots in columns:
     1     2     3     4
Pivots in rows:
     1     2     3     4
U =
    2.0000   -1.0000         0         0
         0    1.5000   -1.0000         0
         0         0    1.3333   -1.0000
         0         0         0    1.2500
```

Again det(A)=det(U)

```
det_A=prod(diag(U))

det_A =
     5
```

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a) The determinant of a, 3 by 3, matrix with rank 1 is obviously zero (since the matrix is singular)

b) The determinant of a, 3 by 3, skew symmetric matrix must be zero because in this case:

$\det(K) = \det(K^T) = \det(-K) = (-1)^3 \det(K) = -\det(K)$

$\Rightarrow 2\det(K) = 0 \Rightarrow \det(K) = 0$.

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a) See (15 b).

b)

```
K=[0 1 0 0; -1 0 0 0; 0 0 0 1; 0 0 -1 0], det_K=det(K)

K =
     0     1     0     0
    -1     0     0     0
     0     0     0     1
     0     0    -1     0
det_K =
     1
```

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Rule 3 and 5 together imply Rule 2.

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```
A=[3 3 4; 6 8 7; -3 5 -9], [P, L, U]=plu(A); U

A =
     3     3     4
     6     8     7
    -3     5    -9
Pivots in columns:
     1     2     3
Pivots in rows:
     1     2     3
U =
     3     3     4
     0     2    -1
     0     0    -1
```

Since no row exchanges have taken place:

$$\det(L) = 1$$

$$\det(U) = -6$$

$$\det(A) = \det(U) = -6$$

$$\det(U^{-1}L^{-1}) = (-\frac{1}{6})(1) = -\frac{1}{6}$$

$$\det(U^{-1}L^{-1}A) = \det(I) = 1$$