Math 323
Linear Algebra and Matrix Theory I
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Key Homework 22

ATLAST Page 142 no: 7

```
A=randint(3), B=randint(4), C=randint(5)

A =
     9     0    -1
    -5     7    -9
     2     5     6
B =
    -1     5     8    -3
     2    -6    -2     6
     6    -2     7    -9
     8     8    -8    -7
C =
    -6    -9    -2     6     0
    -6     5     7    -9     4
     2    -1     0     3    -1
    -4     8    -6    -2    -4
    -6    -1     3     6    -6
```

a)

```
pa=round(poly(A)), ta=trace(A), da=det(A), pb=round(poly(B)),
tb=trace(B), db=det(B), pc=round(poly(C)), tc=trace(C), dc=det(C),

pa =
     1   -22   206  -822
ta =
    22
da =
   822
pb =
           1           7        -201       -1147        6764
tb =
    -7
db =
        6764
pc =
           1           9          92         714         502       -9076
tc =
    -9
dc =
        9076
```

First observe that in MATLAB the leading coefficient of the characteristic polynomial is always 1. (as in plus1)

The coefficients displayed are actually the coeffients of $\det(\lambda I - A)$ rather than the coefficients of $\det(A - \lambda I)$. Under these circumstances the trace of A will equal (-1) times the coefficient of λ^{n-1} in the characteristic polynomial and the determinant will equal $(-1)^n$ times the constant term in the same characteristic polynomial.

b)

```
ea=eig(A), sum_a=sum(ea), prod_a=prod(ea), eb=eig(B), sum_b=sum(eb),
prod_b=prod(eb), ec=eig(C), sum_c=sum(ec), prod_c=prod(ec)
```

```
ea =
   9.3815
   6.3093 + 6.9147i
   6.3093 - 6.9147i
sum_a =
   22.0000
prod_a =
  8.2200e+002 +5.6843e-014i
eb =
   3.8444
 -11.7598 + 0.7178i
 -11.7598 - 0.7178i
  12.6753
sum_b =
   -7.0000
prod_b =
  6.7640e+003
ec =
   0.1114 + 9.4147i
   0.1114 - 9.4147i
  -5.9670 + 1.4685i
  -5.9670 - 1.4685i
   2.7113
sum_c =
   -9.0000
prod_c =
  9.0760e+003
```

Clearly the sum of the eigenvalues, equals the trace and equals (-1) times the coefficient of λ^{n-1} in the characteristic polynomial.

Similarly, the product of the eigenvalues equals the determinant which equals $(-1)^n$ times the constant coefficient in the characteristic polynomial.

c)

```
pa_A=polyvalm(pa, A), pb_B=polyvalm(pb, B), pc_C=polyvalm(pc, C)
```

```
pa_A =
     0     0     0
     0     0     0
     0     0     0
```

```
pb_B =
     0     0     0     0
     0     0     0     0
     0     0     0     0
     0     0     0     0
pc_C =
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
```

Observe that if $p(\lambda)$ denotes the characteristic polynomial of a matrix H, then $p(H) = O$, where O denotes the zero matrix. This result is known as the Cayley-Hamilton theorem.

ATLAST Page 142 no: 8

```
S2=randint(2, 2, 10, 1), S3=randint(3, 3, 10, 2), S4=randint(4, 4, 10,
3), S5=randint(5, 5, 10, 4), S6=randint(6, 6, 10, 5), S7=randint(7, 7,
10, 6),
```

```
S2 =
    -2    -2
    -3    -3
S3 =
     0     4     1
     0    -5     1
     0    -2     5
S4 =
    -3    -2     2     3
     5     8    -1    -5
    -4     0     4     4
     2    -5     0     1
S5 =
    -1    -3    -2     0     1
    -2    -5    -9     2    -1
     0    -2     1     1     0
     1     4     6    -3     2
    -2     4    -1    -1     1
S6 =
     3     4     2    -6     0     1
     0    -2     3    -1    -4     1
    -4    -3     0     5     0    -2
     0    -2     4    -1    -5     2
    -2    -3    -3     4     0    -1
     0     3     2    -4    -2     1
S7 =
    -1     4     2    -2     1    -1     0
     4     0     6     0     2     4     2
     3    -4     2     0    -3    -2     4
     0     0    -3     0     3    -3     1
    -3     6     3    -3     0     0    -1
     0     1     3    -1     6     7     1
     1     0     0    -3     0    -3     1
```

```
p2=round(poly(S2)), p3=round(poly(S3)), p4=round(poly(S4)),
p5=round(poly(S5)), p6=round(poly(S6)), p7=round(poly(S7))

p2 =
     1     5     0
p3 =
     1     0   -23     0
p4 =
     1   -10    -8   102     0
p5 =
     1     7   -24     7     0     0
p6 =
     1    -1     3   -22  -173   -66     0
p7 =
  Columns 1 through 6
           1          -9          29        -120        2197       -3167
  Columns 7 through 8
       -8823           0
```

b)

Theorem:

If the matrix A is singular, then

- A has an eigenvalue 0.
- The constant term in the characteristic polynomial of A equals zero.

Proof.

- If A is singular, then the nullspace of A contains non-zero vectors, and therefore 0 is an eigenvalue of A.
- If 0 is an eigenvalue of A, then 0 is a root of the characteristic polynomial of A, and therefore the constant term of that characteristic polynomial must be 0.

Corollary:

If a matrix A is non-singular then zero is **NOT** a root of its characteristic equation.