

Key Homework 3

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$$\begin{array}{l} 2x + 3y = 1 \\ 10x + 9y = 11 \end{array} \left| \begin{array}{l} 2x + 3y = 1 \\ -6y = 6 \end{array} \right| \Rightarrow y = -1 \Rightarrow 2x - 3 = 1 \Rightarrow 2x = 4 \Rightarrow x = 2.$$

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-1/2 times the first equation needs to be subtracted from the second equation.

$$\begin{array}{l} 2x - 4y = 6 \\ -x + 5y = 0 \end{array} \left| \begin{array}{l} 2x - 4y = 6 \\ 3y = 3 \end{array} \right| \Rightarrow y = 1 \Rightarrow 2x - 4 = 6 \Rightarrow 2x = 10 \Rightarrow x = 5$$

If the right side changes to (-6, 0) then the new result is given by:

$$\begin{array}{l} 2x - 4y = -6 \\ -x + 5y = 0 \end{array} \left| \begin{array}{l} 2x - 4y = -6 \\ 3y = -3 \end{array} \right| \Rightarrow y = -1 \Rightarrow 2x + 4 = -6 \Rightarrow 2x = -10 \Rightarrow x = -5$$

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$$\begin{array}{l} 2x + by = 13 \\ 4x + 8y = g \end{array} \left| \begin{array}{l} 2x + by = 13 \\ (8 - 2b)y = g - 26 \end{array} \right| \Rightarrow \text{the system becomes singular when}$$

$8 - 2b = 0 \Rightarrow b = 4$. It then becomes solvable when $g = 26$.

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$$\begin{array}{l} ax + 3y = -3 \\ 4x + 6y = 6 \end{array}$$

Gaussian elimination will fail permanently if $a = 2$, and fail temporarily if $a = 0$. In the last case we can change the equations.

$$\begin{array}{l} 4x + 6y = 6 \\ 3y = -3 \end{array} \left| \begin{array}{l} 4x + 6y = 6 \\ 3y = -3 \end{array} \right| \Rightarrow y = -1 \Rightarrow 4x - 6 = 6 \Rightarrow 4x = 12 \Rightarrow x = 3.$$

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$$3x - 2y = b_1$$

$$6x - 4y = b_2$$

$(6x - 4y)$ equals two times $(3x - 2y)$, therefore this system has a solution only if $b_2 = 2b_1$.

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$$\begin{array}{l} 2x + 3y + z = 1 \\ 4x + 7y + 5z = 7 \\ -2y + 2z = 6 \end{array} \Rightarrow \begin{array}{l} 2x + 3y + z = 1 \\ y + 3z = 5 \\ -2y + 2z = 6 \end{array} \Rightarrow \begin{array}{l} 2x + 3y + z = 1 \\ y + 3z = 5 \\ 8z = 16 \end{array} \Rightarrow z = 2$$

$$\Rightarrow y + 6 = 5 \Rightarrow y = -1 \Rightarrow 2x - 3 + 2 = 12x = 2 \Rightarrow x = 1.$$

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$$\begin{array}{l} 2x - 3y = 3 \\ 4x - 5y + z = 7 \\ 2x - y - 3z = 5 \end{array} \Rightarrow \begin{array}{l} 2x - 3y = 3 \\ y + z = 1 \\ 2y - 3z = 2 \end{array} \Rightarrow \begin{array}{l} 2x - 3y = 3 \\ y + z = 1 \\ -5z = 0 \end{array} \Rightarrow z = 0$$

$$\Rightarrow y = 1 \Rightarrow 2x - 3 = 3 \Rightarrow x = 3.$$