Math 323
Linear Algebra and Matrix Theory I
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Key Homework 4

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a) We find the elinination matrices by applying to the identity matrix exactly those operations which make up the individual steps of the Gaussian elimination applied to the matrix A.

```
A=[1 1 0; 4 6 1; -2 2 0]

A =
     1     1     0
     4     6     1
    -2     2     0

A2=rowcomb(A, 1, 2, -4), E21=rowcomb(eye(3), 1, 2, -4)

A2 =
     1     1     0
     0     2     1
    -2     2     0
E21 =
     1     0     0
    -4     1     0
     0     0     1

A3=rowcomb(A2, 1, 3, 2), E31=rowcomb(eye(3), 1, 3, 2)

A3 =
     1     1     0
     0     2     1
     0     4     0
E31 =
     1     0     0
     0     1     0
     2     0     1

A4=rowcomb(A3, 2, 3, -2), E32=rowcomb(eye(3), 2, 3, -2)

A4 =
     1     1     0
     0     2     1
     0     0    -2
E32 =
     1     0     0
     0     1     0
     0    -2     1
```

Finally we create the matrix $M = E_{32}E_{31}E_{21}$ and check the identity $MA = U$.

```
M=E32*E31*E21, MA=M*A, U=A4

M =
     1     0     0
    -4     1     0
    10    -2     1
MA =
     1     1     0
     0     2     1
     0     0    -2
U =
     1     1     0
     0     2     1
     0     0    -2
```

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```
h=[1; 1; 1]; A=[3*h -5*h 13*h], x=randint(3, 1, 5), A*x

A =
     3    -5    13
     3    -5    13
     3    -5    13
x =
     5
     3
    -4
ans =
   -52
   -52
   -52
```

Observe that just one pivot will be created during Gaussian elimination as is demonstrated below.

```
A1=rowcomb(A, 1, 2, -1), A2=rowcomb(A1, 1, 3, -1)

A1 =
     3    -5    13
     0     0     0
     3    -5    13
A2 =
     3    -5    13
     0     0     0
     0     0     0
```

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a)

```
for i=1:3 for j=1:3 A(i,j)=2*i-3*j; end; end; A
```

```
A =
    -1    -4    -7
     1    -2    -5
     3     0    -3
```

The step that destroys the zero in the (3, 2) position is the the one which creates a zero in the (3, 1) position. Here is the computation.

```
A1=rowcomb(A, 1, 2, 1), A2=rowcomb(A1, 1, 3, 3)
```

```
A1 =
    -1    -4    -7
     0    -6   -12
     3     0    -3
A2 =
    -1    -4    -7
     0    -6   -12
     0   -12   -24
```

```
A3=rowcomb(A2, 2, 3, -2), E32=rowcomb(eye(3), 2, 3, -2)
```

```
A3 =
    -1    -4    -7
     0    -6   -12
     0     0     0
E32 =
     1     0     0
     0     1     0
     0    -2     1
```

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a) … each one is E times a column of B.
b)

```
h=[3 -7 11]; B=[h; h; h], E=rowcomb(eye(3), 1, 3, -5), EB=E*B
```

```
B =
     3    -7    11
     3    -7    11
     3    -7    11
E =
     1     0     0
     0     1     0
    -5     0     1
EB =
     3    -7    11
     3    -7    11
   -12    28   -44
```

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```
A=[1 2 3 1; 2 3 4 2; 3 5 7 6]
```

```
A =
     1     2     3     1
     2     3     4     2
     3     5     7     6
```

A1=rowcomb(A, 1, 2, -2), A2=rowcomb(A1, 1, 3, -3)

```
A1 =
     1     2     3     1
     0    -1    -2     0
     3     5     7     6
A2 =
     1     2     3     1
     0    -1    -2     0
     0    -1    -2     3
```

A3=rowcomb(A2, 2, 3, -1)

```
A3 =
     1     2     3     1
     0    -1    -2     0
     0     0     0     3
```

Clearly the last equation 0 = 3 has no solution. If we change the last component of the right hand side to a 3, then the last equation will become 0 = 0, and the new system will have a solution. In fact it will have infinitely many solutions. This is shown below.

A(3, 4)=3, A1=rowcomb(A, 1, 2, -2), A2=rowcomb(A1, 1, 3, -3),
A3=rowcomb(A2, 2, 3, -1)

```
A =
     1     2     3     1
     2     3     4     2
     3     5     7     3
A1 =
     1     2     3     1
     0    -1    -2     0
     3     5     7     3
A2 =
     1     2     3     1
     0    -1    -2     0
     0    -1    -2     0
A3 =
     1     2     3     1
     0    -1    -2     0
     0     0     0     0
```

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a)
format rat

A=[-1 2 1 0; 0 -2 -1 2; -1 1 0 3]

```
A =
    -1           2           1           0
     0          -2          -1           2
    -1           1           0           3
```

```
A1=rowscale(A, 1, -1), A2=rowcomb(A1, 1, 3, 1)
```

```
A1 =
     1          -2          -1           0
     0          -2          -1           2
    -1           1           0           3
A2 =
     1          -2          -1           0
     0          -2          -1           2
     0          -1          -1           3
```

```
A3=rowscale(A2, 2, -1/2), A4=rowcomb(A3, 2, 1, 2), A5=rowcomb(A4, 2, 3,
1)
```

```
A3 =
     1          -2          -1           0
     0           1          1/2         -1
     0          -1          -1           3
A4 =
     1           0           0          -2
     0           1          1/2         -1
     0          -1          -1           3
A5 =
     1           0           0          -2
     0           1          1/2         -1
     0           0         -1/2          2
```

```
A6=rowscale(A5, 3, -2), A7=rowcomb(A6, 3, 2, -1/2)
```

```
A6 =
     1           0           0          -2
     0           1          1/2         -1
     0           0           1          -4
A7 =
     1           0           0          -2
     0           1           0           1
     0           0           1          -4
```

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a) We aim to find coefficients a_3, a_2, a_1 and a_0 such that $\sum_{i=1}^{n} i^2 = a_3 n^3 + a_2 n^2 + a_1 n + a_0$. We create a system of four linear equations an four unknowns by choosing n = 0, 1, 2, and 3 respectively. The vander function will quickly create the coefficient matric involved.

```
aug=[vander([0;1; 2; 3]) [0; 1; 5; 14]], answer=rref(aug)
```

```
aug =
     0              0              0              1              0
     1              1              1              1              1
     8              4              2              1              5
    27              9              3              1             14
answer =
     1              0              0              0            1/3
     0              1              0              0            1/2
     0              0              1              0            1/6
     0              0              0              1              0
```

Clearly $a_3 = \frac{1}{3}, a_2 = \frac{1}{2}, a_1 = \frac{1}{6}$, and $a_0 = 0$.

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a)
We solve the given system and create the desired magic square.

```
format short
A=[1 1 1 0 0 0 12; 0 0 0 1 1 1 12; 1 0 0 1 0 0 4; 0 1 0 0 1 0 11; 0 0 1
0 0 1 9; 0 1 0 0 0 1 4; 0 1 0 1 0 0 9], answer=rref(A)
```

```
A =
     1     1     1     0     0     0    12
     0     0     0     1     1     1    12
     1     0     0     1     0     0     4
     0     1     0     0     1     0    11
     0     0     1     0     0     1     9
     0     1     0     0     0     1     4
     0     1     0     1     0     0     9
answer =
     1     0     0     0     0     0    -1
     0     1     0     0     0     0     4
     0     0     1     0     0     0     9
     0     0     0     1     0     0     5
     0     0     0     0     1     0     7
     0     0     0     0     0     1     0
     0     0     0     0     0     0     0
```

The resulting magic square is given by:

```
MagicSquare=[8 1 3; answer(1:3, 7)'; answer(4:6, 7)']
```

```
MagicSquare =
     8     1     3
    -1     4     9
     5     7     0
```