Math 323
Linear Algebra and Matrix Theory I
Fall 1999

Dr. Constant J. Goutziers
Department of Mathematical Sciences
goutzicj@oneonta.edu

Key Homework 5

Strang Page 59 no: 1

c) ABD is 3 by 1
d) AC is 3 by one, while BD is 5 by one. therefore the sum AC + BD is not defined.

Strang Page 59 no: 2

b) (row 1 of A) times (matrix B) equals (row 1 of AB)

Strang Page 59 no: 3

```
A=[1 5; 2 3]; B=[0 2; 0 1]; C=[3 1; 0 0]; ans1=A*B+A*C, ans2=A*(B+C)

ans1 =
     3     8
     6     9
ans2 =
     3     8
     6     9
```

Strang Page 59 no: 7

a) True
b) False, a counter example follows.

```
A=randint(3, 3, 5), h=[1 2 3], B=[h; 9 1 5; h], A*B

A =
    -3     2     1
    -1    -3    -4
     5     4    -3
h =
     1     2     3
B =
     1     2     3
     9     1     5
     1     2     3
ans =
    16    -2     4
   -32   -13   -30
    38     8    26
```

c) True
d) False, a counter example follows.

```
A=randint(3, 3, 5); B=randint(3, 3, 5); ans1=(A*B)^2, ans2=A^2*B^2

ans1 =
         451        -142         276
         131         -11         247
         701        -284        2050
ans2 =
         901         -61         747
         388         -52         258
        1289         319         915
```

Strang Page 59 no: 12

$$AB = BA \Rightarrow \begin{matrix} a & 0 \\ c & 0 \end{matrix} = \begin{matrix} a & b \\ 0 & 0 \end{matrix} \Rightarrow b = c = 0.$$

$$AC = CA \Rightarrow \begin{matrix} 0 & a \\ 0 & c \end{matrix} = \begin{matrix} c & d \\ 0 & 0 \end{matrix} \Rightarrow c = 0 \quad and \quad a = d.$$

This implies that A is a multiple of I.

Strang Page 59 no: 17

a)
```
A=[2 -1; 3 -2]; B=[1 0 4; 1 0 6]; column_2_of_AB=A*B(:, 2)

column_2_of_AB =
     0
     0
```
b)
```
row_2_of_AB=A(2,: )*B

row_2_of_AB =
     1     0     0
```
c)
```
row_2_of_AA=A(2,: )*A

row_2_of_AA =
     0     1
```
d)
```
row_2_of_AAA=A(2, : )*A*A

row_2_of_AAA =
     3    -2
```

Strang Page 59 no: 23

a)
```
A=[0 1; -1 0], A^2
```

```
A =
     0     1
    -1     0
ans =
    -1     0
     0    -1
```

b)

```
B=[0 1; 1 0], C=[0 1; -1 0], ans1=B*C, ans2=-C*B

B =
     0     1
     1     0
C =
     0     1
    -1     0
ans1 =
    -1     0
     0     1
ans2 =
    -1     0
     0     1
```

Strang Page 59 no: 30

The matrices E_{21} and E_{31} are the elimination matrices associated with the first two steps of the Gaussian elimination process for this coefficient matrix. They are:

```
E21=[1 0 0; 1 1 0; 0 0 1], E31=[1 0 0; 0 1 0; -4 0 1]

E21 =
     1     0     0
     1     1     0
     0     0     1
E31 =
     1     0     0
     0     1     0
    -4     0     1
E=E31*E21, A=[2 1 0; -2 0 1; 8 5 3]; EA=E*A

E =
     1     0     0
     1     1     0
    -4     0     1
EA =
     2     1     0
     0     1     1
     0     1     3
```

Strang Page 59 no: 31

```
c=[-2; 8], a=2, b=[1 0], D=[0 1; 5 3], D-c*b/a

c =
    -2
     8
```

```
a =
     2
b =
     1     0
D =
     0     1
     5     3
ans =
     1     1
     1     3
```

This result nicely coincides with the right bottom 2 by 2 submatrix of EA.

ATLAST Page 16 no: 2

a)

`A=[25 20 19; 22 31 34; 19 33 26],  ans_a=sum(A)`

```
A =
    25    20    19
    22    31    34
    19    33    26
ans_a =
    66    84    79
```

b)

`ans_b=ans_a/3`

```
ans_b =
   22.0000   28.0000   26.3333
```

c)

`q=[120 95 87], ans_c=q-ans_a`

```
q =
   120    95    87
ans_c =
    54    11     8
```

d)

`p=[23; 28; 35], ans_d=A*p`

```
p =
    23
    28
    35
ans_d =
        1800
        2564
        2271
```

e)

`ans_e=ans_a(3)*p(3)`

```
ans_e =
        2765
```

ATLAST Page 16 no: 3

`A=[70 50; 30 60], B=[25 40 20; 15 30 25], AB=A*B`

```
A =
    70    50
    30    60
B =
    25    40    20
    15    30    25
AB =
        2500        4300        2650
        1650        3000        2100
```

$AB_{i\,j}$ represents the total consumption by breed i (i = 1 for chimpanzees and i = 2 for gibbons) of food product j (j = 1 for protein, j = 2 carbohydrates, and j = 3 for fat)

ATLAST Page 16 no: 6

A=triu(rand(5)), B=triu(rand(5)), AB=A*B

```
A =
    0.9501    0.7621    0.6154    0.4057    0.0579
         0    0.4565    0.7919    0.9355    0.3529
         0         0    0.9218    0.9169    0.8132
         0         0         0    0.4103    0.0099
         0         0         0         0    0.1389
B =
    0.2028    0.0153    0.4186    0.8381    0.5028
         0    0.7468    0.8462    0.0196    0.7095
         0         0    0.5252    0.6813    0.4289
         0         0         0    0.3795    0.3046
         0         0         0         0    0.1897
AB =
    0.1927    0.5836    1.3659    1.3845    1.4169
         0    0.3409    0.8022    0.9035    1.0154
         0         0    0.4841    0.9760    0.8289
         0         0         0    0.1557    0.1268
         0         0         0         0    0.0263
```

It appears that the product of two upper triangular matrices is again upper triangular. This result can be proved straight from the definition of matrix multiplication.

Proof.
Suppose A and B are two, n by n, upper triangular matrices. This implies that $a_{i\,j} = 0$ if $i > j$, and the same is true for $b_{i\,j}$. We now show that if C = AB, then also $c_{i\,j} = 0$ when $i > j$.

Suppose $i > j$.

$$c_{i\,j} = \sum_{k=1}^{n} a_{i\,k} b_{k\,j} = \sum_{k=1}^{i-1} a_{i\,k} b_{k\,j} + \sum_{k=i}^{n} a_{i\,k} b_{k\,j} = S_1 + S_2$$

Observe that for S_1, $i > k$ and therefore $a_{i\,k} = 0$. This implies that $S_1 = 0$.
Also observe that in S_2, $k \geq i > j$ and therefore $b_{k\,j} = 0$. This implies that $S_2 = 0$ and completes the proof.