Math 323
Linear Algebra and Matrix Theory I
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Key Homework 6

Strang Page 73 no: 1c

c)

```
Cinv=1/(3*7-4*5)*[7 -4; -5 3]

Cinv =
     7    -4
    -5     3
```

Strang Page 84 no: 2

a)

P^{-1} =

```
     0     0     1
     0     1     0
     1     0     0
```

b)

P^{-1} =

```
     0     0     1
     1     0     0
     0     1     0
```

Strang Page 84 no: 3

Perform Gauss Jordan elimination.

```
format rat
a1=rref([10 20 1; 20 50 0])

a1 =
      1             0            1/2
      0             1           -1/5
```

a = ½, c = -1/5.

```
a2=rref([10 20 0; 20 50 1])
```

```
a2 =
      1             0           -1/5
      0             1            1/10
```

b= -1/5, d=1/10

Strang Page 84 no: 6

a)
If AB = AC and A is invertible then A^{-1} A B = A^{-1} A C → B = C.

b)

```
B=[0 0; 0 0], C=[ 1 0; -1 0]

B =
      0             0
      0             0
C =
      1             0
     -1             0
```

Strang Page 84 no: 7

a)
If we substract the first and the seond equation from the third equation, we obtain 0 = -1.

b) $b_3 - b_2 - b_1 = 0$.

c) It turns into a row of zeros.

Strang Page 84 no: 9

```
A=[0 0 0 2; 0 0 3 0; 0 4 0 0; 5 0 0 0], aug=rref([A, eye(4)]);
Ainv=aug(:,5:8)

A =
      0             0             0             2
      0             0             3             0
      0             4             0             0
      5             0             0             0
Ainv =
      0             0             0            1/5
      0             0            1/4            0
      0            1/3            0             0
     1/2            0             0             0


B=[3 2 0 0; 4 3 0 0; 0 0 6 5; 0 0 7 6], aug=rref([B, eye(4)]);
Binv=aug(:,5:8)
```

```
B =
      3              2              0              0
      4              3              0              0
      0              0              6              5
      0              0              7              6
Binv =
      3             -2              0              0
     -4              3              0              0
      0              0              6             -5
      0              0             -7              6
```

Strang Page 84 no: 10

a)

```
A=eye(2), B=-eye(2)

A =
      1              0
      0              1
B =
     -1              0
      0             -1
```

b)

```
A=[1 0; 0 0], B=[ 0 0; 0 1]

A =
      1              0
      0              0
B =
      0              0
      0              1
```

Strang Page 84 no: 11

If C = A B and C is invertible, then $C C^{-1} = A B C^{-1}$ -> $I = A (B C^{-1})$ -> $A^{-1} = B C^{-1}$.

Strang Page 84 no: 14

If A has a column of zeros then also BA has a column of zeros, therefore A cannot be invertible.

Strang Page 84 no: 19

```
aug=rref([[4 -1 -1 -1; -1 4 -1 -1; -1 -1 4 -1; -1 -1 -1 4], eye(4)]);
Ainv=aug(:, 5:8)

Ainv =
     2/5            1/5            1/5            1/5
     1/5            2/5            1/5            1/5
     1/5            1/5            2/5            1/5
     1/5            1/5            1/5            2/5
```

a=2/5, b=1/5

Strang Page 84 no: 21

a)

```
a1=[1 3 1 0; 2 7 0 1]

a1 =
     1     3     1     0
     2     7     0     1

a2=rowcomb(a1, 1, 2, -2)

a2 =
     1     3     1     0
     0     1    -2     1

a3=rowcomb(a2, 2, 1, -3)

a3 =
     1     0     7    -3
     0     1    -2     1
```

b)

```
a1=[1 3 1 0; 3 8 0 1]

a1 =
     1     3     1     0
     3     8     0     1

a2=rowcomb(a1, 1, 2, -3)

a2 =
     1     3     1     0
     0    -1    -3     1

a3=rowcomb(a2, 2, 1, 3)

a3 =
     1     0    -8     3
     0    -1    -3     1

a4=rowscale(a3, 2, -1)

a4 =
     1     0    -8     3
     0     1     3    -1
```

Strang Page 84 no: 24

a)

```
format short
aug=rref([[2 1 1; 1 2 1; 1 1 2], eye(3)])
```

```
aug =
    1.0000         0         0    0.7500   -0.2500   -0.2500
         0    1.0000         0   -0.2500    0.7500   -0.2500
         0         0    1.0000   -0.2500   -0.2500    0.7500
```

```
Ainv=aug(:, 4:6)
```

```
Ainv =
    0.7500   -0.2500   -0.2500
   -0.2500    0.7500   -0.2500
   -0.2500   -0.2500    0.7500
```

a)

```
aug=rref([[2 -1 -1; -1 2 -1; -1 -1 2], eye(3)])
```

```
aug =
    1.0000         0   -1.0000         0   -0.3333   -0.6667
         0    1.0000   -1.0000         0    0.3333   -0.3333
         0         0         0    1.0000    1.0000    1.0000
```

The given matrix is not invertible because it does not have a full set of pivots.

Strang Page 84 no: 29

A is not invertible if c = 0 (a row of zeros), if c = 2 (two equal rows), and c = 7 (two equal columns)

ATLAST Page 21 no: 14

a)

```
A=[3 2; 4 3], B=[3 -2; -4 3], AB=A*B, BA=B*A
```

```
A =
     3     2
     4     3
B =
     3    -2
    -4     3
AB =
     1     0
     0     1
BA =
     1     0
     0     1
```

Since AB = I and BA = I, $A = B^{-1}$ and $B = A^{-1}$.

b)

```
A=[1 0 1; 1 1 0; -1 2 -2], B=[-2 2 -1; 2 -1 1; 3 -2 1], AB=A*B, BA=B*A
```

```
A =
     1     0     1
     1     1     0
    -1     2    -2
```

```
B =
    -2     2    -1
     2    -1     1
     3    -2     1
AB =
     1     0     0
     0     1     0
     0     0     1
BA =
     1     0     0
     0     1     0
     0     0     1
```

Since AB = I and BA = I, A = B^{-1} and B = A^{-1}.

c)

A=[0 0 0 0 1; 1 0 0 0 0; 0 1 0 0 0; 0 0 0 1 0 ; 0 0 1 0 0], B=[0 1 0 0 0; 0 0 1 0 0; 0 0 0 0 1; 0 0 0 1 0; 1 0 0 0 0], AB=A*B, BA=B*A

```
A =
     0     0     0     0     1
     1     0     0     0     0
     0     1     0     0     0
     0     0     0     1     0
     0     0     1     0     0
B =
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     0     1
     0     0     0     1     0
     1     0     0     0     0
AB =
     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     1     0
     0     0     0     0     1
BA =
     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     1     0
     0     0     0     0     1
```

Since AB = I and BA = I, A = B^{-1} and B = A^{-1}.

d)

A=[1 1 1; 1 1 0], B=[1 1 ; -1 0; 1 -1], AB=A*B

```
A =
     1     1     1
     1     1     0
B =
     1     1
    -1     0
     1    -1
```

```
AB =
     1     0
     0     1
```

No it does not follow that A is non-singular and B = A^{-1}, because invertibility and singularity are defined for square matrices only.

ATLAST Page 21 no: 15

a)

```
A=randint(4, 4, 1, 4), Ainv=inv(A)

A =
    -1    -1     0    -1
     0     0    -1    -1
    -1     0     1     0
     1     1     1     0
Ainv =
    0.5000   -0.5000   -1.0000    0.5000
   -1.0000    1.0000    1.0000         0
    0.5000   -0.5000         0    0.5000
   -0.5000   -0.5000         0   -0.5000
```

b)

```
e1=[1;0;0;0]; e2=[0;1;0;0]; e3=[0;0;1;0]; e4=[0;0;0;1];
aug1=rref([A, e1]); x1=aug1(:,5)

x1 =
    0.5000
   -1.0000
    0.5000
   -0.5000

aug2=rref([A, e2]); x2=aug2(:,5)

x2 =
   -0.5000
    1.0000
   -0.5000
   -0.5000

aug3=rref([A, e3]); x3=aug3(:,5)

x3 =
    -1
     1
     0
     0

aug4=rref([A, e4]); x4=aug4(:,5)

x4 =
    0.5000
         0
    0.5000
   -0.5000
```

Because of the column interpretation of matrix multiplication, the vectors x1, x2, x3, and x4 are the columns of A^{-1}.

c)

```
AUG=[A, eye(4)]

AUG =
    -1    -1     0    -1     1     0     0     0
     0     0    -1    -1     0     1     0     0
    -1     0     1     0     0     0     1     0
     1     1     1     0     0     0     0     1

rrefAUG=rref(AUG)

rrefAUG =
  Columns 1 through 7
    1.0000         0         0         0    0.5000   -0.5000   -1.0000
         0    1.0000         0         0   -1.0000    1.0000    1.0000
         0         0    1.0000         0    0.5000   -0.5000         0
         0         0         0    1.0000   -0.5000   -0.5000         0
  Column 8
    0.5000
         0
    0.5000
   -0.5000
```

The inverse of A is formed by the last four columns of the matrix above.

```
inverse_of_A=rrefAUG(:,5:8)

inverse_of_A =
    0.5000   -0.5000   -1.0000    0.5000
   -1.0000    1.0000    1.0000         0
    0.5000   -0.5000         0    0.5000
   -0.5000   -0.5000         0   -0.5000
```

We solved the four equations $A\ x_1 = e_1$, $A\ x_2 = e_2$, $A\ x_3 = e_3$, and $A\ x_4 = e_4$ simultaneously.

ATLAST Page 21 no: 16

a)

```
A=[0.2 0.3 0.4; 0.4 0.4 0.1; 0.5 0.1 0.3]

A =
    0.2000    0.3000    0.4000
    0.4000    0.4000    0.1000
    0.5000    0.1000    0.3000

B=inv(eye(3)-A)

B =
    4.4086    2.6882    2.9032
    3.5484    3.8710    2.5806
    3.6559    2.4731    3.8710
```

Since this inverse matrix exists and has all positive entries, we are sure that for every demand there exist an appropriate production level.

b)

```
A(1, 1)=4.5

A =
    4.5000    0.3000    0.4000
    0.4000    0.4000    0.1000
    0.5000    0.1000    0.3000
```

The third paragraph in the statement of the problem contradicts the second and fourth paragraphs. The fourth paragraph is correct: A**p** represents the amount of chemicals, food, and oil consumed by the production process at production level **p**. This means that a_{ij} represents the number of units of product i necessary for the production of one unit of product j. If we choose $a_{11} = 4.5$ that implies that 4.5 units of product 1 are required to produce one unit of that same product. That is unrealistic.