Math 323
Linear Algebra and Matrix Theory I
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Dr. Constant J. Goutziers
Department of Mathematical Sciences
goutzicj@oneonta.edu

Key Homework 7

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To solve this problem, we perform Gaussian elimination and construct the E matrix.

```
A=[2 1 0; 0 4 2; 6 3 5], U=rowcomb(A, 1, 3, -3)

A =
     2     1     0
     0     4     2
     6     3     5
U =
     2     1     0
     0     4     2
     0     0     5
```

Clearly

```
E=[1 0 0; 0 1 0; -3 0 1], L=inv(E)

E =
     1     0     0
     0     1     0
    -3     0     1
L =
     1     0     0
     0     1     0
     3     0     1
```

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```
A=[2 4; 4 11], [L, U]=slu(A); L, D=diag(diag(U)), U=inv(D)*U

A =
     2     4
     4    11
L =
     1     0
     2     1
D =
     2     0
     0     3
U =
     1     2
     0     1
```

```
B=[1 4 0; 4 12 4; 0 4 0], [L, U]=slu(B); L, D=diag(diag(U)), U=inv(D)*U

B =
     1     4     0
     4    12     4
     0     4     0
L =
     1     0     0
     4     1     0
     0    -1     1
D =
     1     0     0
     0    -4     0
     0     0     4
U =
     1     4     0
     0     1    -1
     0     0     1
```

In both cases U and L are each other's mirror image in the main diagonal. In the next section we will learn that this means that L is the **transpose** of U.

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```
L=[1 0 0; 1 1 0; 1 1 1 ], U=[1 1 1 ; 0 1 1 ; 0 0 1], b=[4; 5; 6], c=L\b,
x=U\c

L =
     1     0     0
     1     1     0
     1     1     1
U =
     1     1     1
     0     1     1
     0     0     1
b =
     4
     5
     6
c =
     4
     1
     1
x =
     3
     0
     1
```

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a) The identity matrix.
b) L^{-1}
c) U

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a)

```
A=[1 1 0; 1 2 1; 0 1 2], [L, U]=slu(A)

A =
     1     1     0
     1     2     1
     0     1     2
L =
     1     0     0
     1     1     0
     0     1     1
U =
     1     1     0
     0     1     1
     0     0     1
```

In this case D = I.

b) A = [a a 0; a a+b b; 0 b b+c], L = [1 0 0; 1 1 0; 0 1 1], D = [a 0 0; 0 b 0; 0 0 c]

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```
[L, U]=slu(pascal(4))

L =
     1     0     0     0
     1     1     0     0
     1     2     1     0
     1     3     3     1
U =
     1     1     1     1
     0     1     2     3
     0     0     1     3
     0     0     0     1
```

```
[L, U]=slu(pascal(5))

L =
     1     0     0     0     0
     1     1     0     0     0
     1     2     1     0     0
     1     3     3     1     0
     1     4     6     4     1
U =
     1     1     1     1     1
     0     1     2     3     4
     0     0     1     3     6
     0     0     0     1     4
     0     0     0     0     1
```

We clearly see Pascal's triangle in the rows of L and the columns of U.

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1a) A is non-singular if x is not equal to zero. Observe that in this case the matrix A can be transformed into the identity matrix, by left multiplication with five elimination matrices. E1 will exchange row 5 and row 4; E2 will exchange row 4 and row 3; E3 will exchange row 3 and row 2; E4 will exchange row 2 and row 1, and finally E5 will scale the first row by a factor 1/x. The result will be that E5*E4*E3*E2*E1*A = I, which immediately shows that A is invertible and that A^{-1} = E5*E4*E3*E2*E1. Observe, we used the result that if a square matrix A has a left inverse, then it is invertible.

1b)

```
format rat
A=[0 1 0 0 0; 0 0 1 0 0; 0 0 0 1 0; 0 0 0 0 1; 13 0 0 0 0], inv(A)


A =
      0         1         0         0         0
      0         0         1         0         0
      0         0         0         1         0
      0         0         0         0         1
     13         0         0         0         0
ans =
      0         0         0         0      1/13
      1         0         0         0         0
      0         1         0         0         0
      0         0         1         0         0
      0         0         0         1         0

A(5,1)=17, inv(A)

A =
      0         1         0         0         0
      0         0         1         0         0
      0         0         0         1         0
      0         0         0         0         1
     17         0         0         0         0
ans =
      0         0         0         0      1/17
      1         0         0         0         0
      0         1         0         0         0
      0         0         1         0         0
      0         0         0         1         0
```

It appears that if x does not equal zero A^{-1} is given by:

$$A^{-1} = \begin{matrix} 0 & 0 & 0 & 0 & 1/x \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{matrix}$$

This claim can easily be justified by computing A A^{-1} .

1c) We experiment with A^5.

```
A, A^5

A =
     0            1            0            0            0
     0            0            1            0            0
     0            0            0            1            0
     0            0            0            0            1
    17            0            0            0            0
ans =
    17            0            0            0            0
     0           17            0            0            0
     0            0           17            0            0
     0            0            0           17            0
     0            0            0            0           17

A(5, 1)=3, A^5

A =
     0            1            0            0            0
     0            0            1            0            0
     0            0            0            1            0
     0            0            0            0            1
     3            0            0            0            0
ans =
     3            0            0            0            0
     0            3            0            0            0
     0            0            3            0            0
     0            0            0            3            0
     0            0            0            0            3
```

It appears that A^5 equals x times the identity matrix. Again this can be proved by executing the product A*A*A*A*A. I recommend that you do not do this by hand! (Maple or Mathematica will do it in a flash) Now we have $A^5 = x I$, which implies $I = A^5 /x$, which in turn can be interpreted as $I^{1/5} = x^{-1/5} A = x^{-1/5} A(x)$. Every new value of x brings a new value of the fifth root of I.

1c) Similarly we expect that the 7 by 7 matrix

```
format short
A=11^(-1/7)*[0 1 0 0 0 0 0; 0 0 1 0 0 0 0 ; 0 0 0 1 0 0 0; 0 0 0 0 1 0
0; 0 0 0 0 0 1 0; 0 0 0 0 0 0 1; 11 0 0 0 0 0 0]
```

```
A =
         0    0.7100         0         0         0         0         0
         0         0    0.7100         0         0         0         0
         0         0         0    0.7100         0         0         0
         0         0         0         0    0.7100         0         0
         0         0         0         0         0    0.7100         0
         0         0         0         0         0         0    0.7100
    7.8095         0         0         0         0         0         0
```

equals a 7-th root of the 7 by 7 identity matrix.

```
A^7
```

```
ans =
     1     0     0     0     0     0     0
     0     1     0     0     0     0     0
     0     0     1     0     0     0     0
     0     0     0     1     0     0     0
     0     0     0     0     1     0     0
     0     0     0     0     0     1     0
     0     0     0     0     0     0     1
```

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1a) Generate the two desired random matrices.

```
A=randint(3, 4, 5), B=randint(4, 5, 5)
```

```
A =
    -1    -1    -3    -5
     5     4     2     2
     0     0     4    -1
B =
     4    -2    -2    -1     0
     0    -3     0     4     4
     2    -3    -4     4     4
    -1     2     2     1     2
```

Compute AB as well as A*B(: , k) for k equals 1 to 5.

```
A*B, A*B(:, 1), A*B(:, 2), A*B(:, 3), A*B(:, 4), A*B(:, 5)
```

```
ans =
    -5     4     4   -20   -26
    22   -24   -14    21    28
     9   -14   -18    15    14
```

```
ans =
    -5
    22
     9
ans =
     4
   -24
   -14
ans =
     4
   -14
   -18
ans =
   -20
    21
    15
ans =
   -26
    28
    14
```

Clearly the last five results are exactly the columns of the matrix AB. This is commensurate with the column interpretation of the product AB.

1b) Similarly we can illustrate the row interpretation of the matrix product AB.

A*B, A(1, :)*B, A(2, :)*B, A(3, :)*B

```
ans =
    -5     4     4   -20   -26
    22   -24   -14    21    28
     9   -14   -18    15    14
ans =
    -5     4     4   -20   -26
ans =
    22   -24   -14    21    28
ans =
     9   -14   -18    15    14
```

1c) First construct a random column vector of length 4.

x=randint(4, 1, 5)

```
x =
     3
     2
    -2
    -2
```

Then illustrate the fact that A**x** represents a linear combination of the columns of A with coefficients given by the vector **x**.

A*x, x(1)*A(:, 1)+x(2)*A(:, 2)+x(3)*A(:, 3)+x(4)*A(:, 4)

```
ans =
    11
    15
    -6
ans =
    11
    15
    -6
```