Math 323
Linear Algebra and Matrix Theory I
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Key Homework 9

ATLAST Page 8 no: 3

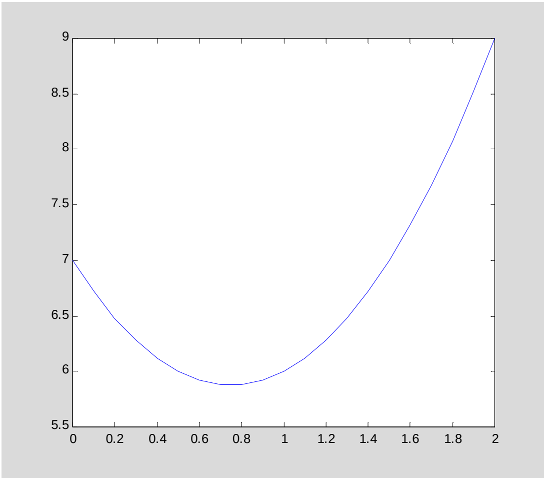
a)
To find the coefficients of the quadratic polynomial through the points (0, 7), (1, 6) and (2, 9), we code the coefficient matrix and the right hand side of the resulting system A**p** = **y** and solve for the coefficient vector **p**.

```
A=[0 0 1; 1 1 1; 4 2 1], y=[7;6; 9], p=A\y

A =
     0     0     1
     1     1     1
     4     2     1
y =
     7
     6
     9
p =
     2
    -3
     7
```

b)
Now the polynomial $p_1x^2 + p_2x + p_3 = 2x^2 - 3x + 7$ can be plotted on the interval [0, 2] by first coding the x-values to be plotted, using the command u = [0 : 0.1 : 2] (this means 0, 0.1, 0.2, 0.3 …1.8, 1.9, 2) and then creating the corresponding y-values, using the command **v = polyval(p, u)**. The command **plot(u, v)** will generate the graph.

```
u=[0: 0.1: 2]; v=polyval(p, u); plot(u, v)
```



c)
To find the coefficients of the cubic polynomial, through the points (-2, 6), (1, 4), (2, 3) and (3, -2) we follow the same technique as under (a). We can save a little work by realizing that the coefficient matrix of the resulting system is a **vandermonde** matrix, that is a matrix whose columns are powers of each other. For instance the third to last column has elements which are the squares of elements of the second to last column of the **vandermonde** matrix. If the vector **x** holds the x-coordinates of the given points and the vector **y** holds the corresponding y-coordinates, then the coefficient vector **p** is readily computed.

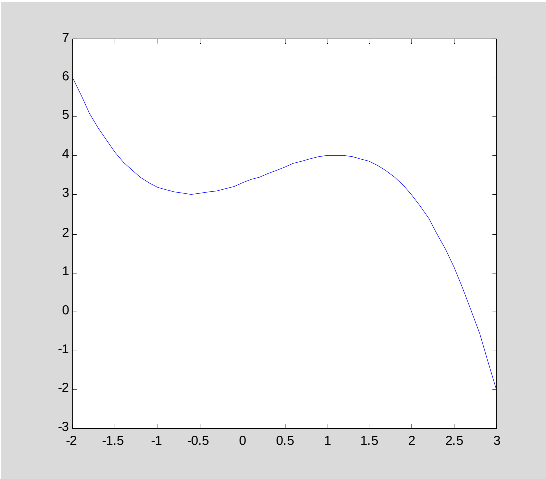
```
format rat
x=[-2; 1; 2; 3];   y=[6; 4; 3; -2]; A=vander(x); p=A\y

p =
   -23/60
     3/10
    47/60
    33/10
```

So $p_1x^3 + p_2x^2 + p_3x + p_4 = -\frac{23}{60}x^3 + \frac{3}{10}x^2 + \frac{47}{60}x + \frac{33}{10}$.

d)
The graph of this third degree polynomial can be generated in a way similar to the one demonstrated under (b).

```
x=[-2:0.1:3]; y=polyval(p, x); plot(x, y)
```



e)
The slope of the graph is given by the first derivative of the represented polynomial. Keeping this in mind the coefficient matrix and right-hand side of the system of equations can be readily determined.

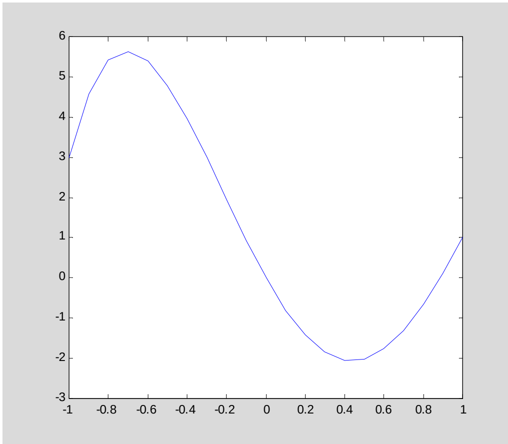
Let $P(x) = ax^4 + bx^3 + cx^2 + dx + e \Rightarrow P'(x) = 4ax^3 + 3bx^2 + 2cx + d$, then the equations are $P(0) = 0, P(1) = 1, P(-1) = 3, P'(-1) = 20$ and $P'(1) = 9$. Observe that the desired coefficients are the coefficients of a, b, c, d, and e (**Not** the coefficients of x^4, x^3, x^2, x and 1)

```
A=[0 0 0 0 1; 1 1 1 1 1; 1 -1 1 -1 1; -4 3 -2 1 0; 4 3 2 1 0], b=[0; 1;
3; 20; 9], p=A\b
```

```
A =
      0            0            0            0            1
      1            1            1            1            1
      1           -1            1           -1            1
     -4            3           -2            1            0
      4            3            2            1            0
b =
      0
      1
      3
     20
      9
p =
   -19/4
    31/4
    27/4
   -35/4
      0
```

f)

```
u=[-1:0.1:1]; v=polyval(p, u); plot(u, v)
```



g)

Observe we have four conditions: $P(0)=100, P(120)=10, P'(0)=0$ and $P'(120)=\tan(\pi/6)=\frac{1}{\sqrt{3}}$. We will use a cubic polynomial $P(x)=p_1x^3+p_2x^2+p_3x+p_4, P'(x)=3p_1x^2+2p_2x+p_3$.

```
format short
A=[0 0 0 1; 120^3 120^2 120 1; 0 0 1 0; 3*120^2 2*120 1 0], b=[100; 10;
0; 1/sqrt(3)], p=A\b

A =
           0           0           0           1
     1728000       14400         120           1
           0           0           1           0
       43200         240           1           0
b =
  100.0000
   10.0000
         0
    0.5774
p =
    0.0001
   -0.0236
         0
  100.0000
```

h)

```
u=[0:1:120]; v=polyval(p, u); plot(u,v)
```