Math 323
Linear Algebra and Matrix Theory I
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Dr. Constant J. Goutziers
Department of Mathematical Sciences
goutzicj@oneonta.edu

Lesson 3
Elimination using Matrices

3.1 Elimination using Matrices

- Example 3.1.1
  Use step by step elimination to solve the system: 2x + 3y - 5z = 4, 4x + 6y + 11z = 5, 5x - 7y - 4z =13.

Code the augmented matrix and use the **rowcomb**, **rowswap** and **rowscale** commands to perform the elimination process. Since no irrationals are involved it is illustrative to use the rational format.

- **rowcomb(A, i, j, l)** will add **l** times the **i-th** row to the **j-th** row of **A**
- **rowswap(A, i, j)** will swap the **i-th** and th **j-th** rows of **A**.
- **rowscale(A, i, l)** will multiply the **i-th** row of **A** by **l**.

```
format rational, aug=[2 3 -5 4; 4 6 11 5; 5 -7 -4 13]

aug =
       2              3             -5              4
       4              6             11              5
       5             -7             -4             13
```

Multiply the first row by 1/ 2 .

```
aug2=rowscale(aug, 1, 1/ 2)

aug2 =
       1            3/2           -5/2              2
       4              6             11              5
       5             -7             -4             13
```

Multiply the first row by -4 and add to the second row.

```
aug3=rowcomb(aug2, 1, 2, -4)
```

```
aug3 =
      1           3/2         -5/2          2
      0            0           21          -3
      5           -7           -4          13
```

Multiply the first row by -5 and add to the third row.

```
aug4=rowcomb(aug3, 1, 3, -5)

aug4 =
      1           3/2         -5/2          2
      0            0           21          -3
      0         -29/2         17/2          3
```

Swap the second and the third row.

```
aug5=rowswap(aug4, 2, 3)

aug5 =
      1           3/2         -5/2          2
      0         -29/2         17/2          3
      0            0           21          -3
```

Multiply the second row -2/29.

```
aug6=rowscale(aug5, 2, -2/29)

aug6 =
      1           3/2         -5/2          2
      0            1         -17/29       -6/29
      0            0           21          -3
```

Multiply the second row by -3/2 and add to the first row.

```
aug7=rowcomb(aug6, 2, 1, -3/2)

aug7 =
      1            0         -47/29       67/29
      0            1         -17/29       -6/29
      0            0           21          -3
```

Multiply the third row by 1/21.

```
aug8=rowscale(aug7, 3, 1/21)

aug8 =
      1            0         -47/29       67/29
      0            1         -17/29       -6/29
      0            0            1         -1/7
```

Multiply the third row by 47/29 and add to the first row.

```
aug9=rowcomb(aug8, 3, 1, 47/29)
```

```
aug9 =
      1             0             *          422/203
      0             1          -17/29         -6/29
      0             0             1           -1/7
```

The star indicates that the (1,3) element of aug9 is very small but not positively identifiable as a zero. Let us take a look.

```
format short, aug9, format rat
```

```
aug9 =
    1.0000         0    -0.0000    2.0788
         0    1.0000    -0.5862   -0.2069
         0         0     1.0000   -0.1429
```

For our purposes a star will mean zero. We now finish the elimination process by multiplying the third row by 17/29 and adding it to the second row.

```
aug10=rowcomb(aug9, 3, 2, 17/29)
```

```
aug10 =
      1             0             *          422/203
      0             1             0          -59/203
      0             0             1           -1/7
```

The solution to the original system is now immediately clear: **x = 422/203, y = -59/203 and z = -1/7**. The matrix **aug10** is said to be in **Reduced Row Echelon** form.

Actually the **Reduced Row Echelon** of a given matrix is unique (we will prove that at a later time), and **MATLAB** contains a command that will immediately produce the **Reduced Row Echelon** form of a matrix. That command, named **rref**, is illustrated below.

```
aug, rref(aug)
```

```
aug =
      2             3            -5             4
      4             6            11             5
      5            -7            -4            13
ans =
      1             0             0          422/203
      0             1             0          -59/203
      0             0             1           -1/7
```

You will probably realize that **rref** is a very powerful tool and we will make heavy use of it in the remaining part of the course.