Key Homework 20

Math 277, Fall 2005

Ordinary Differential Equations

Dr. Goutziers

Initializations

```
> restart;
>
```

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```
> deq:=diff(y(t), t$2)-2*diff(y(t), t)+y(t)=t^(-1)*exp(t);
```

$$deq := \left(\frac{d^2}{dt^2}\mathrm{y}(t)\right) - 2\left(\frac{d}{dt}\mathrm{y}(t)\right) + \mathrm{y}(t) = \frac{\mathbf{e}^t}{t}$$

To find a particular solution to this nonhomogeneous differential equation, we first compute two linearly independent solutions of the corresponding homogeneous equation.

```
> deqh:=lhs(deq)=0;
  aux:=simplify(eval(subs(y(t)=exp(r*t), deqh))/exp(r*t));
  evals:=solve(aux, r);
```

$$deqh := \left(\frac{d^2}{dt^2}\mathrm{y}(t)\right) - 2\left(\frac{d}{dt}\mathrm{y}(t)\right) + \mathrm{y}(t) = 0$$

$$aux := r^2 - 2\,r + 1 = 0$$

$$evals := 1, 1$$

We conclude that

```
> Y[1]:=exp(evals[1]*t);
  Y[2]:=t*Y[1];
```

$$Y_1 := \mathbf{e}^t$$

$$Y_2 := t\,\mathbf{e}^t$$

are two linearly independent solutions of the homogeneous equation, and the general solution of the homogeneous equation is given by.

```
> ygh:=add(c[k]*Y[k], k=1..2);
```

$$ygh := c_1\,\mathbf{e}^t + c_2\,t\,\mathbf{e}^t$$

By the method of variation of parameters, a trial solution for the inhomogeneous equation takes the form

```
> ytry:=add(v[k](t)*Y[k], k=1..2);
```

$$ytry := v_1(t)\,\mathbf{e}^t + v_2(t)\,t\,\mathbf{e}^t$$

```
> ytryp:=diff(ytry, t);
```

$$ytryp := \left(\frac{d}{dt}v_1(t)\right)\mathbf{e}^t + v_1(t)\,\mathbf{e}^t + \left(\frac{d}{dt}v_2(t)\right)t\,\mathbf{e}^t + v_2(t)\,\mathbf{e}^t + v_2(t)\,t\,\mathbf{e}^t$$

Form the first equation for v'_1 and v'_2.

```
> eq1:=add(coeff(ytryp, diff(v[k](t), t))*diff(v[k](t), t), k=1..2)=0;
```

$$eq1 := \left(\frac{d}{dt}v_1(t)\right)\mathbf{e}^t + \left(\frac{d}{dt}v_2(t)\right)t\,\mathbf{e}^t = 0$$

Update the derivative of the trial solution accordingly and compute the second derivative.

```
> ytryp:=ytryp-lhs(eq1);
  ytrypp:=diff(ytryp, t);
```

$$ytryp := v_1(t)\,\mathbf{e}^t + v_2(t)\,\mathbf{e}^t + v_2(t)\,t\,\mathbf{e}^t$$

$$ytrypp := \left(\frac{d}{dt} v_1(t)\right) \mathbf{e}^t + v_1(t)\, \mathbf{e}^t + \left(\frac{d}{dt} v_2(t)\right) \mathbf{e}^t + 2\, v_2(t)\, \mathbf{e}^t + \left(\frac{d}{dt} v_2(t)\right) t\, \mathbf{e}^t + v_2(t)\, t\, \mathbf{e}^t$$

Substitute the formulas for the trial solution and its first two derivatives into the differential equation and thus arrive at a second equation for v'_1 and v'_2.

```
> eq2:=collect(eval(subs({y(t)=ytry, diff(y(t), t)=ytryp, diff(y(t), t$2)=ytrypp},
  deq)), diff);
```

$$eq2 := \left(\frac{d}{dt} v_1(t)\right) \mathbf{e}^t + (\mathbf{e}^t + t\, \mathbf{e}^t) \left(\frac{d}{dt} v_2(t)\right) = \frac{\mathbf{e}^t}{t}$$

Compute v'_1 and v'_2.

```
> val_vp:=solve({eq1, eq2}, {diff(v[1](t), t), diff(v[2](t), t)});
```

$$val\_vp := \{\frac{d}{dt} v_2(t) = \frac{1}{t}, \frac{d}{dt} v_1(t) = -1\}$$

Integrate both equations and find $v_1(t)$ and $v_2(t)$.

```
> for k to 2 do
  rr||k:=map(int, val_vp[k], t);
  od;
```

$$rr1 := v_2(t) = \ln(t)$$

$$rr2 := v_1(t) = -t$$

The general solution of the inhomogeneous equation is given by

```
> yg:=ygh+simplify(subs({rr1, rr2}, ytry));
```

$$yg := c_1\, \mathbf{e}^t + c_2\, t\, \mathbf{e}^t + t\, \mathbf{e}^t (-1 + \ln(t))$$

```
>
```

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```
> deq:=diff(y(t), t$2)+2*diff(y(t), t)+y(t)=exp(-t);
```

$$deq := \left(\frac{d^2}{dt^2} y(t)\right) + 2 \left(\frac{d}{dt} y(t)\right) + y(t) = \mathbf{e}^{(-t)}$$

Observe that since the differential family of $\mathbf{e}^{(-t)}$ is finite, this problem can be solved with the (simpler) method of undetermined coefficients. However the method of variation of parameters will work as well.

To find a particular solution to this nonhomogeneous differential equation, we first compute two linearly independent solutions of the corresponding homogeneous equation.

```
> deqh:=lhs(deq)=0;
  aux:=simplify(eval(subs(y(t)=exp(r*t), deqh))/exp(r*t));
  evals:=solve(aux, r);
```

$$deqh := \left(\frac{d^2}{dt^2} y(t)\right) + 2 \left(\frac{d}{dt} y(t)\right) + y(t) = 0$$

$$aux := r^2 + 2\, r + 1 = 0$$

$$evals := -1, -1$$

We conclude that

```
> Y[1]:=exp(evals[1]*t);
  Y[2]:=t*Y[1];
```

$$Y_1 := \mathbf{e}^{(-t)}$$

$$Y_2 := t\, \mathbf{e}^{(-t)}$$

are two linearly independent solutions of the homogeneous equation, and the general solution of the homogeneous equation is given by.

```
> ygh:=add(c[k]*Y[k], k=1..2);
```

$$ygh := c_1\, \mathbf{e}^{(-t)} + c_2\, t\, \mathbf{e}^{(-t)}$$

By the method of variation of parameters, a trial solution for the inhomogeneous equation takes the form

```
> ytry:=add(v[k](t)*Y[k], k=1..2);
```

$$ytry := v_1(t)\, \mathbf{e}^{(-t)} + v_2(t)\, t\, \mathbf{e}^{(-t)}$$

```
> ytryp:=diff(ytry, t);
```

$$ytryp := \left(\frac{d}{dt} v_1(t)\right) e^{(-t)} - v_1(t)\, e^{(-t)} + \left(\frac{d}{dt} v_2(t)\right) t\, e^{(-t)} + v_2(t)\, e^{(-t)} - v_2(t)\, t\, e^{(-t)}$$

Form the first equation for v'_1 and v'_2.

```
> eq1:=add(coeff(ytryp, diff(v[k](t), t))*diff(v[k](t), t), k=1..2)=0;
```

$$eq1 := \left(\frac{d}{dt} v_1(t)\right) e^{(-t)} + \left(\frac{d}{dt} v_2(t)\right) t\, e^{(-t)} = 0$$

Update the derivative of the trial solution accordingly and compute the second derivative.

```
> ytryp:=ytryp-lhs(eq1);
  ytrypp:=diff(ytryp, t);
```

$$ytryp := -v_1(t)\, e^{(-t)} + v_2(t)\, e^{(-t)} - v_2(t)\, t\, e^{(-t)}$$

$$ytrypp := -\left(\frac{d}{dt} v_1(t)\right) e^{(-t)} + v_1(t)\, e^{(-t)} + \left(\frac{d}{dt} v_2(t)\right) e^{(-t)} - 2\, v_2(t)\, e^{(-t)} - \left(\frac{d}{dt} v_2(t)\right) t\, e^{(-t)} + v_2(t)\, t\, e^{(-t)}$$

Substitute the formulas for the trial solution and its first two derivatives into the differential equation and thus arrive at a second equation for v'_1 and v'_2.

```
> eq2:=collect(eval(subs({y(t)=ytry, diff(y(t), t)=ytryp, diff(y(t), t$2)=ytrypp},
  deq)), diff);
```

$$eq2 := -\left(\frac{d}{dt} v_1(t)\right) e^{(-t)} + \left(e^{(-t)} - t\, e^{(-t)}\right)\left(\frac{d}{dt} v_2(t)\right) = e^{(-t)}$$

Compute v'_1 and v'_2.

```
> val_vp:=solve({eq1, eq2}, {diff(v[1](t), t), diff(v[2](t), t)});
```

$$val\_vp := \left\{\frac{d}{dt} v_2(t) = 1, \frac{d}{dt} v_1(t) = -t\right\}$$

Integrate both equations and find $v_1(t)$ and $v_2(t)$.

```
> for k to 2 do
  rr||k:=map(int, val_vp[k], t);
  od;
```

$$rr1 := v_2(t) = t$$

$$rr2 := v_1(t) = -\frac{t^2}{2}$$

The general solution of the inhomogeneous equation is given by

```
> yg:=ygh+simplify(subs({rr1, rr2}, ytry));
```

$$yg := c_1\, e^{(-t)} + c_2\, t\, e^{(-t)} + \frac{1}{2} t^2\, e^{(-t)}$$

```
>
```

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```
> deq:=diff(y(t), t$2)+y(t)=tan(t)^2;
```

$$deq := \left(\frac{d^2}{dt^2} y(t)\right) + y(t) = \tan(t)^2$$

To find a particular solution to this nonhomogeneous differential equation, we first compute two linearly independent solutions of the corresponding homogeneous equation.

```
> deqh:=lhs(deq)=0;
  aux:=simplify(eval(subs(y(t)=exp(r*t), deqh))/exp(r*t));
  evals:=solve(aux, r);
```

$$deqh := \left(\frac{d^2}{dt^2} y(t)\right) + y(t) = 0$$

$$aux := r^2 + 1 = 0$$

$$evals := I, -I$$

We conclude that

```
> Y[1]:=cos(Im(evals[1])*t);
```

```
Y[2]:=sin(Im(evals[1])*t);
```

$$Y_1 := \cos(t)$$

$$Y_2 := \sin(t)$$

are two linearly independent solutions of the homogeneous equation, and the general solution of the homogeneous equation is given by.

```
> ygh:=add(c[k]*Y[k], k=1..2);
```

$$ygh := c_1 \cos(t) + c_2 \sin(t)$$

By the method of variation of parameters, a trial solution for the inhomogeneous equation takes the form

```
> ytry:=add(v[k](t)*Y[k], k=1..2);
```

$$ytry := v_1(t)\cos(t) + v_2(t)\sin(t)$$

```
> ytryp:=diff(ytry, t);
```

$$ytryp := \left(\frac{d}{dt} v_1(t)\right)\cos(t) - v_1(t)\sin(t) + \left(\frac{d}{dt} v_2(t)\right)\sin(t) + v_2(t)\cos(t)$$

Form the first equation for v'_1 and v'_2.

```
> eq1:=add(coeff(ytryp, diff(v[k](t), t))*diff(v[k](t), t), k=1..2)=0;
```

$$eq1 := \left(\frac{d}{dt} v_1(t)\right)\cos(t) + \left(\frac{d}{dt} v_2(t)\right)\sin(t) = 0$$

Update the derivative of the trial solution accordingly and compute the second derivative.

```
> ytryp:=ytryp-lhs(eq1);
  ytrypp:=diff(ytryp, t);
```

$$ytryp := -v_1(t)\sin(t) + v_2(t)\cos(t)$$

$$ytrypp := -\left(\frac{d}{dt} v_1(t)\right)\sin(t) - v_1(t)\cos(t) + \left(\frac{d}{dt} v_2(t)\right)\cos(t) - v_2(t)\sin(t)$$

Substitute the formulas for the trial solution and its first two derivatives into the differential equation and thus arrive at a second equation for v'_1 and v'_2.

```
> eq2:=collect(eval(subs({y(t)=ytry, diff(y(t), t)=ytryp, diff(y(t), t$2)=ytrypp},
  deq)), diff);
```

$$eq2 := -\left(\frac{d}{dt} v_1(t)\right)\sin(t) + \left(\frac{d}{dt} v_2(t)\right)\cos(t) = \tan(t)^2$$

Compute v'_1 and v'_2.

```
> val_vp:=solve({eq1, eq2}, {diff(v[1](t), t), diff(v[2](t), t)});
```

$$val\_vp := \left\{\frac{d}{dt} v_2(t) = \frac{\sin(t)^2}{\cos(t)}, \frac{d}{dt} v_1(t) = -\frac{\sin(t)^3}{\cos(t)^2}\right\}$$

Integrate both equations and find $v_1(t)$ and $v_2(t)$.

```
> for k to 2 do
  rr||k:=map(int, val_vp[k], t);
  od;
```

$$rr1 := v_2(t) = -\sin(t) + \ln(\sec(t) + \tan(t))$$

$$rr2 := v_1(t) = -\frac{\sin(t)^4}{\cos(t)} - \sin(t)^2\cos(t) - 2\cos(t)$$

The general solution of the inhomogeneous equation is given by

```
> yg:=ygh+simplify(subs({rr1, rr2}, ytry));
```

$$yg := c_1\cos(t) + c_2\sin(t) - 2 + \sin(t)\ln\left(\frac{1+\sin(t)}{\cos(t)}\right)$$

```
>
```

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```
> deq:=diff(y(theta), theta$2)+y(theta)=sec(theta)^3;
```

$$deq := \left(\frac{d^2}{d\theta^2} y(\theta)\right) + y(\theta) = \sec(\theta)^3$$

To find a particular solution to this nonhomogeneous differential equation, we first compute two linearly independent solutions of the corresponding homogeneous equation.

```
> deqh:=lhs(deq)=0;
  aux:=simplify(eval(subs(y(theta)=exp(r*theta), deqh))/exp(r*theta));
  evals:=solve(aux, r);
```

$$deqh := \left(\frac{d^2}{d\theta^2}\, y(\theta)\right) + y(\theta) = 0$$

$$aux := r^2 + 1 = 0$$

$$evals := I, -I$$

We conclude that

```
> Y[1]:=cos(Im(evals[1])*theta);
  Y[2]:=sin(Im(evals[1])*theta);
```

$$Y_1 := \cos(\theta)$$

$$Y_2 := \sin(\theta)$$

are two linearly independent solutions of the homogeneous equation, and the general solution of the homogeneous equation is given by.

```
> ygh:=add(c[k]*Y[k], k=1..2);
```

$$ygh := c_1 \cos(\theta) + c_2 \sin(\theta)$$

By the method of variation of parameters, a trial solution for the inhomogeneous equation takes the form

```
> ytry:=add(v[k](theta)*Y[k], k=1..2);
```

$$ytry := v_1(\theta)\cos(\theta) + v_2(\theta)\sin(\theta)$$

```
> ytryp:=diff(ytry, theta);
```

$$ytryp := \left(\frac{d}{d\theta}\, v_1(\theta)\right)\cos(\theta) - v_1(\theta)\sin(\theta) + \left(\frac{d}{d\theta}\, v_2(\theta)\right)\sin(\theta) + v_2(\theta)\cos(\theta)$$

Form the first equation for v'_1 and v'_2.

```
> eq1:=add(coeff(ytryp, diff(v[k](theta), theta))*diff(v[k](theta), theta), k=1..2)=0;
```

$$eq1 := \left(\frac{d}{d\theta}\, v_1(\theta)\right)\cos(\theta) + \left(\frac{d}{d\theta}\, v_2(\theta)\right)\sin(\theta) = 0$$

Update the derivative of the trial solution accordingly and compute the second derivative.

```
> ytryp:=ytryp-lhs(eq1);
  ytrypp:=diff(ytryp, theta);
```

$$ytryp := -v_1(\theta)\sin(\theta) + v_2(\theta)\cos(\theta)$$

$$ytrypp := -\left(\frac{d}{d\theta}\, v_1(\theta)\right)\sin(\theta) - v_1(\theta)\cos(\theta) + \left(\frac{d}{d\theta}\, v_2(\theta)\right)\cos(\theta) - v_2(\theta)\sin(\theta)$$

Substitute the formulas for the trial solution and its first two derivatives into the differential equation and thus arrive at a second equation for v'_1 and v'_2.

```
> eq2:=collect(eval(subs({y(theta)=ytry, diff(y(theta), theta)=ytryp, diff(y(theta),
  theta$2)=ytrypp}, deq)), diff);
```

$$eq2 := -\left(\frac{d}{d\theta}\, v_1(\theta)\right)\sin(\theta) + \left(\frac{d}{d\theta}\, v_2(\theta)\right)\cos(\theta) = \sec(\theta)^3$$

Compute v'_1 and v'_2.

```
> val_vp:=solve({eq1, eq2}, {diff(v[1](theta), theta), diff(v[2](theta), theta)});
```

$$val\_vp := \left\{\frac{d}{d\theta}\, v_2(\theta) = \frac{1}{\cos(\theta)^2},\ \frac{d}{d\theta}\, v_1(\theta) = -\frac{\sin(\theta)}{\cos(\theta)^3}\right\}$$

Integrate both equations and find $v_1(\theta)$ and $v_2(\theta)$.

```
> for k to 2 do
  rr||k:=map(int, val_vp[k], theta);
  od;
```

$$rr1 := v_2(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$rr2 := v_1(\theta) = -\frac{1}{2}\,\frac{1}{\cos(\theta)^2}$$

The general solution of the inhomogeneous equation is given by

```
> yg:=ygh+simplify(subs({rr1, rr2}, ytry));
```

$$yg := c_1 \cos(\theta) + c_2 \sin(\theta) - \frac{1}{2}\frac{-1 + 2\cos(\theta)^2}{\cos(\theta)}$$

```
>
```

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Like in many variation of parameters problems, the integrals in this problem are particularly unpleasant. In order to make sure that we are able to obtain a real solution, we declare the variable *t* to be real.

```
> interface(showassumed=0);
  assume(t, real);
```

1

```
> deq:=1/2*diff(y(t), t$2)+2*y(t)=tan(2*t)-1/2*exp(t);
```

$$deq := \frac{1}{2}\left(\frac{d^2}{dt^2}\,y(t)\right) + 2\,y(t) = \tan(2\,t) - \frac{1}{2}\,e^t$$

To find a particular solution to this nonhomogeneous differential equation, we first compute two linearly independent solutions of the corresponding homogeneous equation.

```
> deqh:=lhs(deq)=0;
  aux:=simplify(eval(subs(y(t)=exp(r*t), deqh))/exp(r*t));
  evals:=solve(aux, r);
```

$$deqh := \frac{1}{2}\left(\frac{d^2}{dt^2}\,y(t)\right) + 2\,y(t) = 0$$

$$aux := \frac{r^2}{2} + 2 = 0$$

$$evals := 2\,I, -2\,I$$

We conclude that

```
> Y[1]:=cos(Im(evals[1])*t);
  Y[2]:=sin(Im(evals[1])*t);
```

$$Y_1 := \cos(2\,t)$$

$$Y_2 := \sin(2\,t)$$

are two linearly independent solutions of the homogeneous equation, and the general solution of the homogeneous equation is given by.

```
> ygh:=add(c[k]*Y[k], k=1..2);
```

$$ygh := c_1 \cos(2\,t) + c_2 \sin(2\,t)$$

By the method of variation of parameters, a trial solution for the inhomogeneous equation takes the form

```
> ytry:=add(v[k](t)*Y[k], k=1..2);
```

$$ytry := v_1(t)\cos(2\,t) + v_2(t)\sin(2\,t)$$

```
> ytryp:=diff(ytry, t);
```

$$ytryp := \left(\frac{d}{dt}\,v_1(t)\right)\cos(2\,t) - 2\,v_1(t)\sin(2\,t) + \left(\frac{d}{dt}\,v_2(t)\right)\sin(2\,t) + 2\,v_2(t)\cos(2\,t)$$

Form the first equation for v'_1 and v'_2.

```
> eq1:=add(coeff(ytryp, diff(v[k](t), t))*diff(v[k](t), t), k=1..2)=0;
```

$$eq1 := \left(\frac{d}{dt}\,v_1(t)\right)\cos(2\,t) + \left(\frac{d}{dt}\,v_2(t)\right)\sin(2\,t) = 0$$

Update the derivative of the trial solution accordingly and compute the second derivative.

```
> ytryp:=ytryp-lhs(eq1);
  ytrypp:=diff(ytryp, t);
```

$$ytryp := -2\,v_1(t)\sin(2\,t) + 2\,v_2(t)\cos(2\,t)$$

$$ytrypp := -2\left(\frac{d}{dt}\,v_1(t)\right)\sin(2\,t) - 4\,v_1(t)\cos(2\,t) + 2\left(\frac{d}{dt}\,v_2(t)\right)\cos(2\,t) - 4\,v_2(t)\sin(2\,t)$$

Substitute the formulas for the trial solution and its first two derivatives into the differential equation and thus arrive at a second equation for v'_1 and v'_2.

```
> eq2:=collect(eval(subs({y(t)=ytry, diff(y(t), t)=ytryp, diff(y(t), t$2)=ytrypp},
  deq)), diff);
```

$$eq2 := -\left(\frac{d}{dt} v_1(t)\right)\sin(2\,t) + \left(\frac{d}{dt} v_2(t)\right)\cos(2\,t) = \tan(2\,t) - \frac{1}{2}\,\mathbf{e}^t$$

Compute v'_1 and v'_2.

```
> val_vp:=solve({eq1, eq2}, {diff(v[1](t), t), diff(v[2](t), t)});
```

$$val\_vp := \left\{\frac{d}{dt} v_2(t) = \sin(2\,t) - \frac{1}{2}\,\mathbf{e}^t \cos(2\,t),\ \frac{d}{dt} v_1(t) = -\frac{1}{2}\,\frac{(2\sin(2\,t) - \mathbf{e}^t\cos(2\,t))\sin(2\,t)}{\cos(2\,t)}\right\}$$

Integrate both equations and find $v_1(t)$ and $v_2(t)$.

```
> for k to 2 do
  rr||k:=simplify(map(int, val_vp[k], t), trig);
  od;
```

$$rr1 := v_2(t) = -\frac{1}{2}\cos(2\,t) - \frac{1}{10}\,\mathbf{e}^t\cos(2\,t) - \frac{1}{5}\,\mathbf{e}^t\sin(2\,t)$$

$$rr2 := v_1(t) = \frac{\tan(t) - \frac{1}{5}\,\mathbf{e}^t + \frac{1}{5}\,\mathbf{e}^t\tan(t)^2 + \frac{1}{5}\,\mathbf{e}^t\tan(t)}{1+\tan(t)^2} + \frac{1}{2}\ln(\tan(t)-1) - \frac{1}{2}\ln(\tan(t)+1)$$

In order to be sure we obtain a real solution, we take the real part of the result obtained by substituting the formulas for $v_1(t)$ and $v_2(t)$ into the trial solution. The general solution of the inhomogeneous equation now takes the form.

```
> yg:=ygh+Re(combine(simplify(subs({rr1, rr2}, ytry)), trig));
```

$$yg := c_1\cos(2\,t) + c_2\sin(2\,t) - \frac{1}{5}\,\mathbf{e}^t + \frac{1}{2}\cos(2\,t)\ln\left(\left|\frac{\sin(t)-\cos(t)}{\cos(t)}\right|\right) - \frac{1}{2}\cos(2\,t)\ln\left(\left|\frac{\sin(t)+\cos(t)}{\cos(t)}\right|\right)$$

This result is equivalent to the answer given in the textbook.

```
>
```

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```
> deq:=t*diff(y(t), t$2)-(t+1)*diff(y(t), t)+y(t)=t^2;
```

$$deq := t\left(\frac{d^2}{dt^2}\mathrm{y}(t)\right) - (t+1)\left(\frac{d}{dt}\mathrm{y}(t)\right) + \mathrm{y}(t) = t^2$$

Given are two linearly independent solutions of the corresponding homogeneous equation.

```
> Y[1]:=exp(t);
  Y[2]:=t+1;
```

$$Y_1 := \mathbf{e}^t$$

$$Y_2 := t+1$$

We conclude that the general solution of the homogeneous equation is given by

```
> ygh:=add(c[k]*Y[k], k=1..2);
```

$$ygh := c_1\,\mathbf{e}^t + c_2\,(t+1)$$

By the method of variation of parameters, a trial solution for the inhomogeneous equation takes the form

```
> ytry:=add(v[k](t)*Y[k], k=1..2);
```

$$ytry := v_1(t)\,\mathbf{e}^t + v_2(t)\,(t+1)$$

```
> ytryp:=diff(ytry, t);
```

$$ytryp := \left(\frac{d}{dt} v_1(t)\right)\mathbf{e}^t + v_1(t)\,\mathbf{e}^t + \left(\frac{d}{dt} v_2(t)\right)(t+1) + v_2(t)$$

Form the first equation for v'_1 and v'_2.

```
> eq1:=add(coeff(ytryp, diff(v[k](t), t))*diff(v[k](t), t), k=1..2)=0;
```

$$eq1 := \left(\frac{d}{dt} v_1(t)\right)\mathbf{e}^t + \left(\frac{d}{dt} v_2(t)\right)(t+1) = 0$$

Update the derivative of the trial solution accordingly and compute the second derivative.

```
> ytryp:=ytryp-lhs(eq1);
  ytrypp:=diff(ytryp, t);
```

$$ytryp := v_1(t)\,\mathbf{e}^t + v_2(t)$$

$$ytrypp := \left(\frac{d}{dt} v_1(t)\right)\mathbf{e}^t + v_1(t)\,\mathbf{e}^t + \left(\frac{d}{dt} v_2(t)\right)$$

Substitute the formulas for the trial solution and its first two derivatives into the differential equation and thus arrive at a second equation for v'_1 and v'_2.

```
> eq2:=collect(eval(subs({y(t)=ytry, diff(y(t), t)=ytryp, diff(y(t), t$2)=ytrypp},
  deq)), diff);
```

$$eq2 := t\,\mathbf{e}^t\left(\frac{d}{dt} v_1(t)\right) + t\left(\frac{d}{dt} v_2(t)\right) + t\,v_1(t)\,\mathbf{e}^t - (t+1)\,(v_1(t)\,\mathbf{e}^t + v_2(t)) + v_1(t)\,\mathbf{e}^t + v_2(t)\,(t+1) = t^2$$

Compute v'_1 and v'_2.

```
> val_vp:=solve({eq1, eq2}, {diff(v[1](t), t), diff(v[2](t), t)});
```

$$val\_vp := \{\frac{d}{dt} v_1(t) = \frac{t+1}{\mathbf{e}^t}, \frac{d}{dt} v_2(t) = -1\}$$

Integrate both equations and find $v_1(t)$ and $v_2(t)$.

```
> for k to 2 do
  rr||k:=map(int, val_vp[k], t);
  od;
```

$$rr1 := v_1(t) = -\frac{2+t}{\mathbf{e}^t}$$

$$rr2 := v_2(t) = -t$$

A particular solution of the inhomogeneous equation is given by

```
> yp:=simplify(subs({rr1, rr2}, ytry));
```

$$yp := -2 - 2\,t - t^2$$

```
>
```