Key Homework 23

Math 277, Fall 2005

Ordinary Differential Equations

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Initializations

```
> restart;
  with(inttrans):
```

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The Laplace transform of cos(2 t) is defined by the integral

```
> e1:=Int(exp(-s*t)*cos(2*t), t=0..infinity);
```

$$e1 := \int_0^\infty \mathrm{e}^{(-s\,t)} \cos(2\,t)\,dt$$

For this improper integral to converge, the variable s needs to be positive.

```
> assume(s>0);
  ans[5]:=simplify(value(e1));
```

$$ans_5 := \frac{s}{s^2+4}$$

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```
> f:=t->piecewise(0<t and t<1, 1-t, t>1, 0);
```

$$f := t \to \text{piecewise}(0 < t \textbf{ and } t < 1,\, 1-t,\, 1 < t,\, 0)$$

The Laplace transform of f(t) is defined by the integral

```
> e1:=Int(exp(-s*t)*'f'(t), t=0..infinity);
```

$$e1 := \int_0^\infty \mathrm{e}^{(-s\,t)}\, \mathrm{f}(t)\,dt$$

This integral will actually converge for all values of the variable s, because it equals

$$\int_0^1 \mathrm{e}^{(-s\,t)}\, \mathrm{f}(t)\,dt$$

```
> ans[10]:=value(e1);
```

$$ans_{10} := \frac{s-1+\mathrm{e}^{(-s)}}{s^2}$$

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Instead of an old fashioned Laplace transform table, we use Maple's Laplace transform routine.

```
> f:=t->t^3-t*exp(t)+exp(4*t)*cos(t);
```

$$f := t \to t^3 - t\,\mathrm{e}^t + \mathrm{e}^{(4\,t)} \cos(t)$$

```
> ans[15]:=L*['f'(t)]=laplace(f(t), t, s);
```

$$ans_{15} := L\,[\,\mathrm{f}(t)\,] = \frac{6}{s^4} - \frac{1}{(s-1)^2} + \frac{s-4}{(s-4)^2+1}$$

This formula is valid for $s > 4$.

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Use Maple's Laplace transform routine.

```
> f:=t->t^4*exp(5*t)-exp(t)*cos(sqrt(7)*t);
```

$$f := t \to t^4 \mathbf{e}^{(5\,t)} - \mathbf{e}^t \cos(\sqrt{7}\, t)$$

```
> ans[19]:=L*['f'(t)]=laplace(f(t), t, s);
```

$$ans_{19} := L\,[\,\mathrm{f}(t)\,] = \frac{24}{(s-5)^5} - \frac{s-1}{7\left(\frac{(s-1)^2}{7}+1\right)}$$

This formula is valid for $s > 5$.

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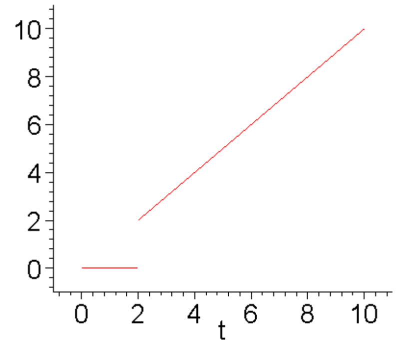
```
> f:=t->piecewise(0<=t and t<2, 0, 2<=t and t<=10, t, undefined);
```

$$f := t \to \mathrm{piecewise}(0 \le t \text{ and } t < 2, 0, 2 \le t \text{ and } t \le 10, t, undefined)$$

Since the function *f* is continuous on the interval [0,10], except for a jump discontinuity at 2, it is piecewise continuous on [0,10].

```
> plot(f(t), t=0..10, discont=true, axes=frame, view=[-1..11, -1..11]);
```



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```
> f:=t->(t^2-t-20)/(t^2+7*t+10);
```

$$f := t \to \frac{t^2 - t - 20}{t^2 + 7\,t + 10}$$

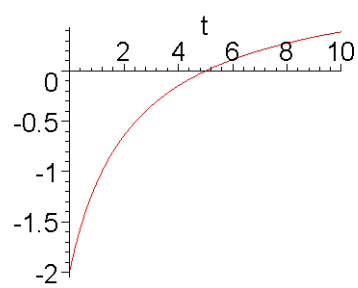
Compute the zeros of the denominator.

```
> zd:=solve(denom(f(t))=0, t);
```

$$zd := -2, -5$$

Since the zeros of the denominator are outside of the interval [0,10], we conclude that the function *f* is continuous on the interval [0,10].

```
> plot(f(t), t=0..10);
```



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It is easy to verify that for all given examples

$$\lim_{s \to \infty} \mathrm{F}(s) = 0$$