Key Homework 24

Math 277, Fall 2005

Ordinary Differential Equations

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Initializations

```
> restart;
  with(inttrans):
  with(student):
```

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Observe

$$L\left[e^{-t}\cos 3t+e^{6t}-1\right](s)=L\left[e^{-t}\cos 3t\right](s)+L\left[e^{6t}\right](s)-L[1](s)$$

$$=L[\cos 3t](s+1)+L[1](s-6)-L[1](s)=\frac{s+1}{(s+1)^2+9}+\frac{1}{s-6}-\frac{1}{s}$$

We verify this computation using Maple's Laplace transform routine.

```
> e1:=laplace(exp(-t)*cos(3*t)+exp(6*t)-1, t, s);
```

$$e1 := \frac{s^3+s^2+6\,s+60}{(s^2+2\,s+10)\,(s-6)\,s}$$

This result can be put into the desired format by performing a partial fraction decomposition followed by a completion of the square with respect to the variable s.

```
> ans[3]:=completesquare(convert(e1, parfrac, s), s);
```

$$ans_3 := \frac{s+1}{(s+1)^2+9}+\frac{1}{s-6}-\frac{1}{s}$$

```
>
```

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Observe

$$L\left[e^{-2t}\sin 2t+e^{3t}t^2\right](s)=L[\sin 2t](s+2)+L\left[t^2\right](s-3)$$

$$=\frac{2}{(s+2)^2+4}+\frac{2!}{(s-3)^3}=\frac{2}{(s+2)^2+4}+\frac{2}{(s-3)^3}$$

We verify this computation using Maple's Laplace transform routine.

```
> e1:=laplace(exp(-2*t)*sin(2*t)+exp(3*t)*t^2, t, s);
```

$$e1 := \frac{2}{s^2+4\,s+8}+\frac{2}{(s-3)^3}$$

This result can be put into the desired format by performing a completion of the square with respect to the variable s.

```
> ans[6]:=completesquare(e1, s);
```

$$ans_6 := \frac{2}{(s+2)^2+4}+\frac{2}{(s-3)^3}$$

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Observe

$$L\left[(t-1)^4\right](s) = L\left[\sum_{k=0}^{4}\binom{4}{k}(t)^k(-1)^{4-k}\right](s)$$

$$= \sum_{k=0}^{4}\binom{4}{k}(-1)^{4-k} L\left[t^k\right](s) = \sum_{k=0}^{4}\binom{4}{k}(-1)^{4-k}\frac{k!}{s^{k+1}}$$

$$= \sum_{k=0}^{4}(-1)^k\frac{4!}{(4-k)!}\frac{1}{s^{k+1}} = \sum_{k=0}^{4}\frac{24(-1)^k}{(4-k)!}\frac{1}{s^{k+1}}$$

$$= \frac{1}{s} - \frac{4}{s^2} + \frac{12}{s^3} - \frac{24}{s^4} + \frac{24}{s^5}$$

We verify this computation using Maple's Laplace transform routine.

```
> e1:=laplace((t-1)^4, t, s);
```

$$e1 := \frac{24 - 24\,s + 12\,s^2 - 4\,s^3 + s^4}{s^5}$$

This result can be put into the desired format by performing a partial fraction decomposition with respect to the variable s.

```
> ans[7]:=sort(convert(e1, parfrac, s));
```

$$ans_7 := \frac{1}{s} - \frac{4}{s^2} + \frac{12}{s^3} - \frac{24}{s^4} + \frac{24}{s^5}$$

```
>
```

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First observe that

$$\sin 3t \cos 3t = \frac{1}{2}\sin 6t$$

so

$$L\left[\sin 3t \cos 3t\right](s) = \frac{1}{2}L\left[\sin 6t\right](s) = \frac{1}{2}\frac{6}{s^2+36} = \frac{3}{s^2+36}$$

We verify this computation using Maple's Laplace transform routine.

```
> e1:=laplace(sin(3*t)*cos(3*t), t, s);
```

$$e1 := \frac{3}{s^2+36}$$

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Observe

$$L\left[e^{at}\cos bt\right](s) = L\left[\cos bt\right](s-a) = \frac{s-a}{(s-a)^2+b^2}$$

We verify this computation using Maple's Laplace transform routine.

```
> e1:=laplace(exp(a*t)*cos(b*t), t, s);
```

$$e1 := \frac{s-a}{(s-a)^2+b^2}$$

```
>
```