Key Homework 26

Math 277, Fall 2005

Ordinary Differential Equations

Dr. Goutziers

Initializations

```
> restart;
  with(inttrans):
  u:=Heaviside;
```

$$u := Heaviside$$

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Code the differential equation and the initial conditions.

```
> deq:=diff(y(t), t$2)+y(t)=t-(t-4)*u(t-2);
  ic:={y(0)=0, D(y)(0)=1};
```

$$deq := \left(\frac{d^2}{dt^2}\mathrm{y}(t)\right) + \mathrm{y}(t) = t - (t-4)\,\mathrm{Heaviside}(t-2)$$

$$ic := \{\mathrm{y}(0) = 0, \mathrm{D}(y)(0) = 1\}$$

Take the Laplace transform of both sides of the differential equation and substitute the initial conditions.

```
> e1:=laplace(deq, t, s);
```

$$e1 := s^2\,\mathrm{laplace}(\mathrm{y}(t), t, s) - \mathrm{D}(y)(0) - s\,\mathrm{y}(0) + \mathrm{laplace}(\mathrm{y}(t), t, s) = \frac{1 + \mathbf{e}^{(-2\,s)}\,(2\,s - 1)}{s^2}$$

```
> e2:=subs(ic, e1);
```

$$e2 := s^2\,\mathrm{laplace}(\mathrm{y}(t), t, s) - 1 + \mathrm{laplace}(\mathrm{y}(t), t, s) = \frac{1 + \mathbf{e}^{(-2\,s)}\,(2\,s - 1)}{s^2}$$

Solve for the Laplace transform of the solution of the initial value problem.

```
> e3:=solve(e2, laplace(y(t), t, s));
```

$$e3 := \frac{s^2 + 1 + 2\,\mathbf{e}^{(-2\,s)}\,s - \mathbf{e}^{(-2\,s)}}{s^2\,(s^2 + 1)}$$

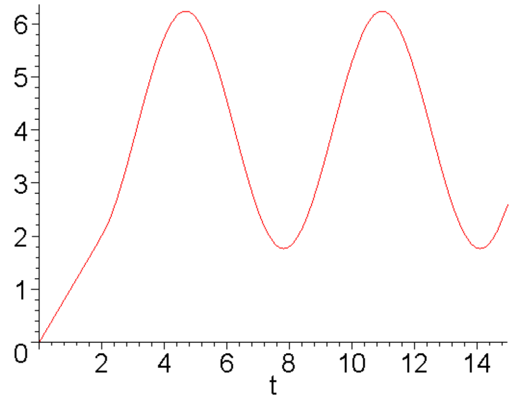
Compute the inverse Laplace transform of this result.

```
> sol:=invlaplace(e3, s, t);
```

$$sol := t - \mathrm{Heaviside}(t-2)\left(-2 - 4\sin\left(\frac{t}{2} - 1\right)^2 - \sin(t-2) + t\right)$$

Plot the solution.

```
> plot(sol, t=0..15);
```



>

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Code the differential equation and the initial conditions.

```
> deq:=diff(y(t), t$2)+4*diff(y(t), t)+4*y(t)=u(t-Pi)-u(t-2*Pi);
  ic:={y(0)=0, D(y)(0)=0};
```

$$deq := \left(\frac{d^2}{dt^2}\mathrm{y}(t)\right) + 4\left(\frac{d}{dt}\mathrm{y}(t)\right) + 4\,\mathrm{y}(t) = \mathrm{Heaviside}(t-\pi) - \mathrm{Heaviside}(t-2\pi)$$

$$ic := \{\mathrm{y}(0)=0, \mathrm{D}(y)(0)=0\}$$

Take the Laplace transform of both sides of the differential equation and substitute the initial conditions.

```
> e1:=laplace(deq, t, s);
```

$$e1 := s^2\,\mathrm{laplace}(\mathrm{y}(t), t, s) - \mathrm{D}(y)(0) - s\,\mathrm{y}(0) + 4\,s\,\mathrm{laplace}(\mathrm{y}(t), t, s) - 4\,\mathrm{y}(0) + 4\,\mathrm{laplace}(\mathrm{y}(t), t, s) = \frac{\mathbf{e}^{(-s\,\pi)} - \mathbf{e}^{(-2\,s\,\pi)}}{s}$$

```
> e2:=subs(ic, e1);
```

$$e2 := s^2\,\mathrm{laplace}(\mathrm{y}(t), t, s) + 4\,s\,\mathrm{laplace}(\mathrm{y}(t), t, s) + 4\,\mathrm{laplace}(\mathrm{y}(t), t, s) = \frac{\mathbf{e}^{(-s\,\pi)} - \mathbf{e}^{(-2\,s\,\pi)}}{s}$$

Solve for the Laplace transform of the solution of the initial value problem.

```
> e3:=solve(e2, laplace(y(t), t, s));
```

$$e3 := \frac{\mathbf{e}^{(-s\,\pi)} - \mathbf{e}^{(-2\,s\,\pi)}}{s\,(s^2+4\,s+4)}$$

Compute the inverse Laplace transform of this result.

```
> sol:=invlaplace(e3, s, t);
```

$$sol := \frac{1}{4}(1 - \mathbf{e}^{(-2\,t+2\,\pi)}(1+2\,t-2\,\pi))\,\mathrm{Heaviside}(t-\pi) + \frac{1}{4}(-1 + \mathbf{e}^{(-2\,t+4\,\pi)}(1+2\,t-4\,\pi))\,\mathrm{Heaviside}(t-2\,\pi)$$

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Code the differential equation and the initial conditions.

```
> deq:=diff(y(t), t$2)+5*diff(y(t), t)+6*y(t)=0+u(t-1)*(t-0)+u(t-5)*(1-t);
  ic:={y(0)=0, D(y)(0)=2};
```

$$deq := \left(\frac{d^2}{dt^2}\mathrm{y}(t)\right) + 5\left(\frac{d}{dt}\mathrm{y}(t)\right) + 6\,\mathrm{y}(t) = \mathrm{Heaviside}(t-1)\,t + \mathrm{Heaviside}(t-5)\,(1-t)$$

$$ic := \{\mathrm{y}(0)=0, \mathrm{D}(y)(0)=2\}$$

Take the Laplace transform of both sides of the differential equation and substitute the initial conditions.

```
> e1:=laplace(deq, t, s);
```

$$e1 := s^2\,\mathrm{laplace}(\mathrm{y}(t), t, s) - \mathrm{D}(y)(0) - s\,\mathrm{y}(0) + 5\,s\,\mathrm{laplace}(\mathrm{y}(t), t, s) - 5\,\mathrm{y}(0) + 6\,\mathrm{laplace}(\mathrm{y}(t), t, s) = \frac{(s+1)\,\mathbf{e}^{(-s)} - \mathbf{e}^{(-5\,s)}(4\,s+1)}{s^2}$$

```
> e2:=subs(ic, e1);
```

$$e2 := s^2\,\text{laplace}(y(t), t, s) - 2 + 5\,s\,\text{laplace}(y(t), t, s) + 6\,\text{laplace}(y(t), t, s) = \frac{(s+1)\,e^{(-s)} - e^{(-5\,s)}\,(4\,s+1)}{s^2}$$

Solve for the Laplace transform of the solution of the initial value problem.

```
> e3:=solve(e2, laplace(y(t), t, s));
```

$$e3 := \frac{2\,s^2 + e^{(-s)}\,s + e^{(-s)} - 4\,e^{(-5\,s)}\,s - e^{(-5\,s)}}{s^2\,(s^2+5\,s+6)}$$

Compute the inverse Laplace transform of this result.

```
> sol:=invlaplace(e3, s, t);
```

$$sol := -2\,e^{(-3\,t)} + 2\,e^{(-2\,t)} - \frac{1}{36}\,\text{Heaviside}(t-5)\,(-11 + 6\,t - 63\,e^{(-2\,t+10)} + 44\,e^{(-3\,t+15)})$$
$$+ \frac{1}{36}\,\text{Heaviside}(t-1)\,(-5 + 6\,t - 9\,e^{(-2\,t+2)} + 8\,e^{(-3\,t+3)})$$

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Code the differential equation and the initial conditions.

```
> deq:=diff(y(t), t$2)+3*diff(y(t), t)+2*y(t)=exp(-t)+u(t-3)*(1-exp(-t));
  ic:={y(0)=2, D(y)(0)=-1};
```

$$deq := \left(\frac{d^2}{dt^2}\,y(t)\right) + 3\left(\frac{d}{dt}\,y(t)\right) + 2\,y(t) = e^{(-t)} + \text{Heaviside}(t-3)\,(1 - e^{(-t)})$$

$$ic := \{y(0) = 2,\ D(y)(0) = -1\}$$

Take the Laplace transform of both sides of the differential equation and substitute the initial conditions.

```
> e1:=laplace(deq, t, s);
```

$$e1 := s^2\,\text{laplace}(y(t), t, s) - D(y)(0) - s\,y(0) + 3\,s\,\text{laplace}(y(t), t, s) - 3\,y(0) + 2\,\text{laplace}(y(t), t, s) = \frac{e^{(-3\,s)}}{s} + \frac{1 - e^{(-3\,s-3)}}{s+1}$$

```
> e2:=subs(ic, e1);
```

$$e2 := s^2\,\text{laplace}(y(t), t, s) - 5 - 2\,s + 3\,s\,\text{laplace}(y(t), t, s) + 2\,\text{laplace}(y(t), t, s) = \frac{e^{(-3\,s)}}{s} + \frac{1 - e^{(-3\,s-3)}}{s+1}$$

Solve for the Laplace transform of the solution of the initial value problem.

```
> e3:=solve(e2, laplace(y(t), t, s));
```

$$e3 := \frac{7\,s^2 + 6\,s + 2\,s^3 + e^{(-3\,s)}\,s + e^{(-3\,s)} - s\,e^{(-3\,s-3)}}{s\,(s^3 + 4\,s^2 + 5\,s + 2)}$$

Compute the inverse Laplace transform of this result.

```
> sol:=invlaplace(e3, s, t);
```

$$sol := \frac{1}{2}\,(1 - 2\,e^{(3-t)} - 2\,e^{(3-2\,t)} + e^{(-2\,t+6)} - 2\,e^{(-t)}\,(t-4))\,\text{Heaviside}(t-3) + e^{(-t)}\,(2+t)$$

```
>
```