**Math 276**
**Calculus III**

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# Lesson 5
## Lines and Planes

# Initializations

```
> restart;
  with(VectorCalculus):
  BasisFormat(false):
>
```

# 5.1 Lines in 3-Space

A line in 3-space is given by a vector equation of the form $\mathbf{r}(t) = \mathbf{r}_0 + t\,\mathbf{v}$. This equation represents a line through the tip of the vector $\mathbf{r}_0$ in the direction of the vector $\mathbf{v}$.

## Examples

### *Example 5.1.1*

Find the vector equation of the line through the point $(5,-1,4)$ and parallel to the vector $\langle 1, 2, 3\rangle$. Compute the point of intersection of this line with the plane $2x + 5y - z = 11$.

**Solution**

The vector equation of the line is given by

$$\mathbf{r}(t) = \langle 5,-1, 4\rangle + t\langle 1, 2, 3\rangle$$

```
> r:=<5, -1, 4>+t*<1, 2, 3>;
```

$$r := \begin{bmatrix} 5+t \\ -1+2t \\ 4+3t \end{bmatrix} \quad (2.1.1.1)$$

In order to compute the point of intersection of this line with the plane $2x + 5y - z = 11$, we substitute the components of $\mathbf{r}$ in this equation and solve for the variable $t$.

```
> plane:=2*x+5*y-z=11;
```

$$plane := 2x + 5y - z = 11 \quad (2.1.1.2)$$

```
> eq_t:=subs({x=r[1], y=r[2], z=r[3]}, plane);
```

$$eq\_t := 1 + 9t = 11 \quad (2.1.1.3)$$

```
> ti:=solve(eq_t, t);
```

$$ti := \frac{10}{9} \quad (2.1.1.4)$$

The point of intersection is obtained by substitution of this $t$-value into the vector equation of the line.

```
> poi:=subs(t=ti, r);
```

$$poi := \begin{bmatrix} \frac{55}{9} \\ \frac{11}{9} \\ \frac{22}{3} \end{bmatrix} \quad (2.1.1.5)$$

```
>
```

# 5.2 Planes in 3-Space

## Examples

### *Example 5.2.1*

Compute the equation of the plane through the point $P = (1,-5,-2)$ and parallel to the vectors $\mathbf{v_1} = \langle 1, 2, 3\rangle$ and $\mathbf{v_2} = \langle 5,-7, 11\rangle$.

**Solution**

Code the point $P$, the vectors $\mathbf{v}_1$ and $\mathbf{v}_2$.

```
> P:=[1, -5, 2];
  v1:=<1, 2, 3>;
  v2:=<5, -7, 11>;
```

$$P := [1, -5, 2]$$

$$v1 := \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$v2 := \begin{bmatrix} 5 \\ -7 \\ 11 \end{bmatrix} \quad (3.1.1.1)$$

The cross product of $\mathbf{v}_1$ and $\mathbf{v}_2$ provides a normal vector for the desired plane.

```
> n:=v1 &x v2;
```

$$n := \begin{bmatrix} 43 \\ 4 \\ -17 \end{bmatrix} \quad (3.1.1.2)$$

The equation of the plane is given by

$$n_1(x-P_x) + n_2(y-P_y) + n_3(z-P_z) = 0$$

```
> plane:=n[1]*(x-P[1])+n[2]*(y-P[2])+n[3]*(z-P[3])=0;
```

$$plane := 43\,x + 11 + 4\,y - 17\,z = 0 \quad (3.1.1.3)$$

The *x*, *y* and *z* in this equation can be ordered using Maple's **sort** command.

```
> sort(plane, [x,y,z]);
```

$$43\,x + 4\,y - 17\,z + 11 = 0 \qquad (3.1.1.4)$$

```
>
```