

## Lesson 7

### Cartesian, Cylindrical and Spherical Coordinates

#### Initializations

```
> restart;  
>
```

#### 7.1 Conversion between coordinate systems.

##### Examples

###### Example 7.1

Find the Cartesian coordinates of the point  $P = (r, \theta, z) = \left(3, \frac{\pi}{6}, 5\right)$ .

###### Solution

Substitute the cylindrical coordinates  $(r, \theta, z) = \left(3, \frac{\pi}{6}, 5\right)$   $([r, \theta, z]) = \left(\left[3, \frac{\pi}{6}, 5\right]\right)$  into the transformation formulas

$$\begin{aligned}x &= r \cos \theta \\y &= r \sin \theta \\z &= z\end{aligned}$$

```
> P:={r=3, theta=Pi/6, z=5};
```

$$P := \left\{r=3, \theta = \frac{1}{6} \pi, z=5\right\} \quad (2.1.1.1)$$

```
> [x,y,z]=simplify(subs(P, [r*cos(theta), r*sin(theta), z]));  
evalf(%);
```

$$\begin{aligned}([x, y, z]) &= \left(\left[\frac{3}{2} \sqrt{3}, \frac{3}{2}, 5\right]\right) \\([x, y, z]) &= ([2.598076212, 1.500000000, 5.])\end{aligned} \quad (2.1.1.2)$$

```
>
```

###### Example 7.2

Find the spherical coordinates of the point  $P = (x, y, z) = (1, 2, 3)$ .

###### Solution

Substitute the Cartesian coordinates  $(x, y, z) = (1, 2, 3)$  into the transformation formulas

$$\begin{aligned}\rho &= \sqrt{x^2 + y^2 + z^2} \\ \theta &= \arctan(y, x)\end{aligned}$$

$$\phi = \arccos\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$

```
> P:={x=1, y=2, z=3};
      P := {x=1, y=2, z=3} (2.1.2.1)
```

```
> [rho, theta, phi]=simplify(subs(P, [sqrt(x^2+y^2+z^2),
arctan(y,x), arccos(z/sqrt(x^2+y^2+z^2))])));
evalf(%);
      ([rho, theta, phi]) = ([sqrt(14), arctan(2), arccos(3/14*sqrt(14))])
      ([rho, theta, phi]) = ([3.741657387, 1.107148718, 0.6405223126]) (2.1.2.2)
```

### Example 7.3

Find the cylindrical coordinates of the point  $P = (\rho, \theta, \phi) = \left(3, \frac{\pi}{3}, \frac{\pi}{6}\right)$ .

#### Solution

Substitute the spherical coordinates  $(\rho, \theta, \phi) = \left(3, \frac{\pi}{3}, \frac{\pi}{6}\right)$  into the transformation formulas

$$r = \rho \sin \phi$$

$$\theta = \theta$$

$$z = \rho \cos \phi$$

```
> P:={rho=3, theta=Pi/3, phi=Pi/6};
      P := {rho=3, theta=1/3 pi, phi=1/6 pi} (2.1.3.1)
```

```
> [r, theta, z]=simplify(subs(P, [rho*sin(phi), theta, rho*
cos(phi)]));
evalf(%);
      ([r, theta, z]) = ([3/2, 1/3 pi, 3/2*sqrt(3)])
      ([r, theta, z]) = ([1.500000000, 1.047197551, 2.598076212]) (2.1.3.2)
```