

Lesson 6

Derivatives as a Rate of Change

Initializations

```
> restart;
```

6.1 Rates of Change

The real importance of derivatives lies in the fact that they represent an instantaneous rate of change. We have already used derivatives to describe instantaneous velocity, but the applications are much wider spread. The purpose of this lesson is to show you how maple can be used to quickly solve rate of change problems.

Examples

Example 6.1.1

Suppose a particle is moving along the x axis and its position at time t is given by $f(t) = 2t^3 - 9t^2 + 12t + 1$. Answer the following questions:

- What is the velocity of the particle at time t ?
- What is the velocity of the particle after three seconds?
- When is the particle at rest?
- When is the particle moving in positive direction?

Solution

- What is the velocity of the particle at time t ?

The velocity at time t is given by the derivative of the function f .

```
> f:=t->2*t^3-9*t^2+12*t+1;
```

$$f := t \rightarrow 2t^3 - 9t^2 + 12t + 1 \quad (2.1.1.1)$$

```
> v:=diff(f(t), t);
```

$$v := 6t^2 - 18t + 12 \quad (2.1.1.2)$$

```
>
```

- What is the velocity of the particle after three seconds?

To find the velocity of the particle after three seconds, we substitute $t = 3$ in the formula for v .

```
> velocity_3:=subs(t=3, v);
```

$$velocity_3 := 12 \quad (2.1.1.3)$$

```
>
```

- When is the particle at rest?

The particle is at rest when its velocity equals zero. Hence, we should solve the equation $v(t) = 0$ for the variable t .

```
> t0:=solve(v=0, t);  
t0 := 2, 1 (2.1.1.4)
```

```
>
```

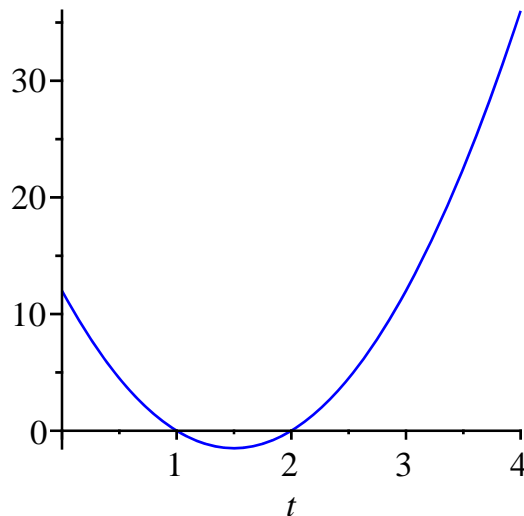
iv) When is the particle moving in positive direction?

The particle is moving in positive direction if $v(t) > 0$. We can solve this problem through an inequality or we can do it graphically.

```
> sol:=solve(v>0, t);  
sol := RealRange(-∞, Open(1)), RealRange(Open(2), ∞) (2.1.1.5)
```

Clearly, the particle moves to the right if $t < 1$ or if $t > 2$. This finding is supported by the graph of the velocity v .

```
> plot(v, t=0..4, color=blue);
```



```
>
```

Example 6.1.2

The quantity of charge Q in Coulombs (C) that has passed through a surface up to time t , measured in seconds, is given by $Q(t) = t^3 - 2t^2 + 6t + 2$. Find the current when $t = 0.5$ sec, and when $t = 1$ sec.

Solution

The current equals the charge passing through the surface per unit of time. We code the function $Q(t)$, take its derivative and substitute the appropriate values for the variable t .

```
> Q:=t->t^3-2*t^2+6*t+2;  
Q := t → t3 - 2t2 + 6t + 2 (2.1.2.1)
```

```
> current:=diff(Q(t), t);  
current := 3t2 - 4t + 6 (2.1.2.2)
```

```
> current_1:=subs(t=0.5, current);  
current_1 := 1.625 (2.1.2.3)
```

```
current_1 := 4.75 (2.1.2.3)
```

```
> current_2 := subs(t=1, current);  
current_2 := 5 (2.1.2.4)
```

Remark: As we have seen earlier, it is possible to obtain the derivative of a Maple function as a Maple function rather than a Maple expression. The latter is advantageous in a situation like this where we have to evaluate the derivative of $Q(t)$ for a variety of values of the variable t . We obtain the current as a Maple function of t by using the **D** routine rather than the **diff** routine.

```
> current := D(Q);  
current := t → 3 t2 - 4 t + 6 (2.1.2.5)
```

```
> current_1 := current(0.5);  
current_1 := 4.75 (2.1.2.6)
```

```
> current_2 := current(1);  
current_2 := 5 (2.1.2.7)
```

```
>
```