Math 173
Calculus I

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Lesson 7
Implicit Differentiation

Initializations

```
> restart;
```

7.1 Implicit Differentiation

Examples

Example 7.1.1

Suppose that the function $y = y(x)$ is defined by the implicit relation $x^2y - 7xy^3 + \sin 5y = 10$. Compute $\frac{dy}{dx}$.

Solution

We enter the expression $x^2 y - 7 x y^3 + \sin(5 y) = 10$ and to instruct the system that y is really a function of x we replace y by $y(x)$ before differentiating.

```
> e1:=x^2*y-7*x*y^3+sin(5*y)=10;
```

$$e1 := x^2 y - 7 x y^3 + \sin(5 y) = 10 \qquad (2.1.1.1)$$

```
> e2:=subs(y=y(x), e1);
```

$$e2 := x^2 y(x) - 7 x y(x)^3 + \sin(5 y(x)) = 10 \qquad (2.1.1.2)$$

```
> e3:=diff(e2, x);
```

$$e3 := 2 x y(x) + x^2 \left(\frac{d}{dx} y(x)\right) - 7 y(x)^3 - 21 x y(x)^2 \left(\frac{d}{dx} y(x)\right) + 5 \cos(5 y(x)) \left(\frac{d}{dx} y(x)\right) = 0 \qquad (2.1.1.3)$$

Now solve this equation for $\frac{dy}{dx}$.

```
> der1:=solve(e3, diff(y(x), x));
```

$$der1 := -\frac{y(x) \left(2 x - 7 y(x)^2\right)}{x^2 - 21 x y(x)^2 + 5 \cos(5 y(x))} \qquad (2.1.1.4)$$

If so desired the $y(x)$ in this formula can be replaced by y.

```
> der1:=subs(y(x)=y, der1);
```

$$der1 := -\frac{y \left(2 x - 7 y^2\right)}{x^2 - 21 x y^2 + 5 \cos(5 y)} \qquad (2.1.1.5)$$

Actually, Maple has a built-in routine for implicit differentiation. This routine will allow you to check your answer.

```
> implicitdiff(e1, y, x);
```

$$-\frac{y\left(2\,x-7\,y^{2}\right)}{x^{2}-21\,x\,y^{2}+5\cos(5\,y)} \qquad \textbf{(2.1.1.6)}$$

```
>
```