Math 173
Calculus I

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Lesson 11
Absolute Extrema

Initializations

```
> restart;
>
```

11.1 Computing Absolute Extrema

To find the absolute extrema of a continuous function f on a closed and bounded interval $[a, b]$, do the following

i. Compute the critical points c of f.
ii. Evaluate $f(c)$ for all the critical points c on the interval (a, b). Then evaluate $f(a)$ and $f(b)$.
iii. The absolute maximum M of f on the interval $[a, b]$, is the maximum of the values computed in Part ii.
iv. The absolute minimum m of f on the interval $[a, b]$, is the minimum of the values computed in Part ii.

Examples

Example 11.1.1

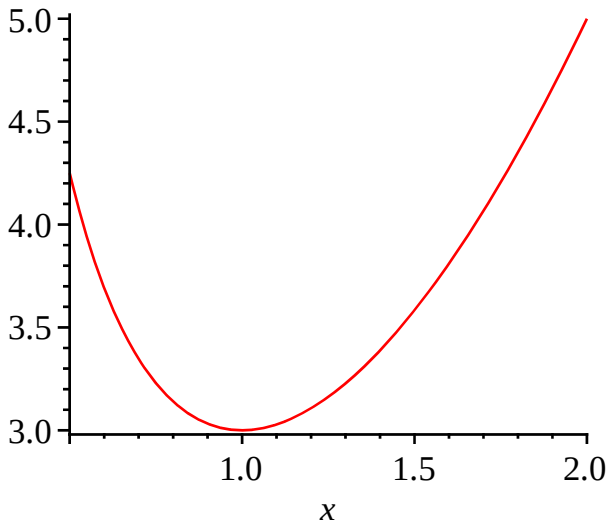
Compute the absolute maximum and the absolute minimum of the function $f(x) = x^2 + \frac{2}{x}$ on the interval $\left[\frac{1}{2}, 2\right]$.

Solution
First, plot the graph of this function.

```
> f:=x->x^2+2/x;
```

$$f := x \to x^2 + \frac{2}{x} \qquad (2.1.1.1)$$

```
> plot(f(x), x=1/2..2);
```



>

Observe that f is continuous on the interval $\left[\frac{1}{2}, 2\right]$. We therefore only need to evaluate the function f at its critical points, and at the endpoints of the domain.

```
> der1:=simplify(diff(f(x), x));
```

$$der1 := \frac{2\left(x^3-1\right)}{x^2} \qquad (2.1.1.2)$$

From this result, we see that the critical points for f are 0 and 1. Since 0 is outside of the interval $\left[\frac{1}{2}, 2\right]$, 1 is the only critical point that remains to be examined. Adding the endpoints of the interval to the candidate list, one obtains

```
> can:=[1/2,1,2];
```

$$can := \left[\frac{1}{2}, 1, 2\right] \qquad (2.1.1.3)$$

The corresponding function values are

```
> fv:=map(f, can);
```

$$fv := \left[\frac{17}{4}, 3, 5\right] \qquad (2.1.1.4)$$

The absolute extrema are found as the minimum m and the maximum M of the values in this list. The values in a Maple list are obtained using the **op** command, as demonstrated below.

```
> m:=min(op(fv));
  M:=max(op(fv));
```

$$m := 3$$

$$M := 5 \qquad (2.1.1.5)$$

It makes good sense to report where the absolute extrema are obtained. The function f has an absolute minimum 3 at 1 and an absolute maximum 5 at 2. Frequently, the extrema are listed as ordered pairs (points). Points are represented in Maple as lists of two elements $[a, b]$.

```
> extr:=[1, f(1)], [2, f(2)];
```

(2.1.1.6)

> *extr* := [1, 3], [2, 5] **(2.1.1.6)**