

Lesson 18

Centers of Mass

Initializations

```
> restart;  
with(plots):
```

18.1 The continuous case

Examples

Example 18.1.1

Find the center of mass (centroid) of the region bounded by $y = 2x^2 - 3$ and $y = 3x - 4$.

Solution

First we sketch the region and find its vertices.

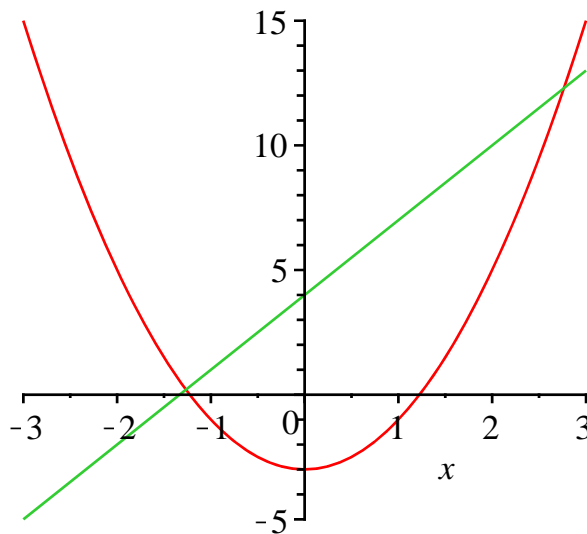
```
> f:=x->2*x^2-3;
```

$$f := x \rightarrow 2x^2 - 3 \quad (2.1.1.1)$$

```
> g:=x->3*x+4;
```

$$g := x \rightarrow 3x + 4 \quad (2.1.1.2)$$

```
> plot({f(x), g(x)}, x=-3..3);
```



```
> pts:=solve(f(x)=g(x), x);
```

$$pts := \frac{3}{4} + \frac{1}{4}\sqrt{65}, \frac{3}{4} - \frac{1}{4}\sqrt{65} \quad (2.1.1.3)$$

Since the location of the centroid is independent of the mass density ρ , we may without loss of generality assume $\rho = 1$. Compute the mass.

```
> Mass:=Int(g(x)-f(x), x=pts[2]..pts[1]);
Mass:=simplify(value(Mass));
```

$$Mass := \int_{\frac{3}{4} - \frac{1}{4}\sqrt{65}}^{\frac{3}{4} + \frac{1}{4}\sqrt{65}} (-2x^2 + 7 + 3x) dx$$

$$Mass := \frac{65}{24} \sqrt{65} \quad (2.1.1.4)$$

Compute the moment about the y axis.

```
> M[y]:=Int(x*(g(x)-f(x)), x=pts[2]..pts[1]);
M[y]:=simplify(value(M[y]));
```

$$M_y := \int_{\frac{3}{4} - \frac{1}{4}\sqrt{65}}^{\frac{3}{4} + \frac{1}{4}\sqrt{65}} x (-2x^2 + 7 + 3x) dx$$

$$M_y := \frac{65}{32} \sqrt{65} \quad (2.1.1.5)$$

Compute the moment about the x axis.

```
> M[x]:=Int(1/2*(g(x)^2-f(x)^2), x=pts[2]..pts[1]);
M[x]:=simplify(value(M[x]));
```

$$M_x := \int_{\frac{3}{4} - \frac{1}{4}\sqrt{65}}^{\frac{3}{4} + \frac{1}{4}\sqrt{65}} \left(\frac{1}{2} (3x+4)^2 - \frac{1}{2} (2x^2-3)^2 \right) dx$$

$$M_x := \frac{65}{8} \sqrt{65} \quad (2.1.1.6)$$

Find the centroid.

```
> centroid:=[M[y]/Mass, M[x]/Mass];
```

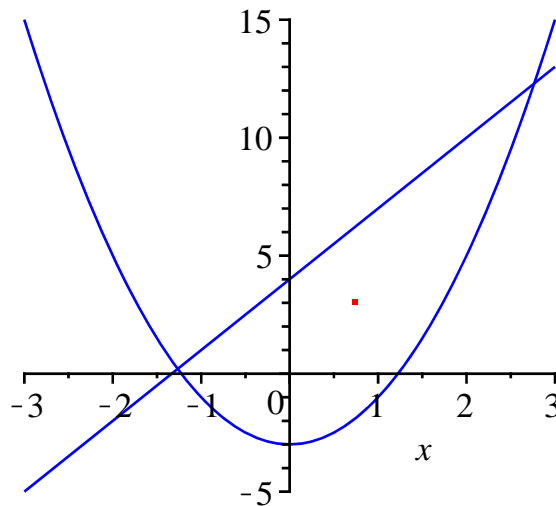
$$centroid := \left[\frac{3}{4}, 3 \right] \quad (2.1.1.7)$$

We can visualize the location of the centroid in the plot of f and g .

```
> p1:=plot({f(x), g(x)}, x=-3..3, color=blue):
```

```
> p2:=pointplot(centroid, style=point, symbol=box, color=
red):
```

```
> display([p1, p2]);
```



>

▼ 18.2 The discrete case

▼ Examples

▼ Example 18.2.1

Find the center of gravity of the points with masses $m_1 = 5$, $m_2 = 7$, and $m_3 = 10$ at locations $(-1, 3)$, $(4, 8)$, and $(10, -5)$ respectively.

Solution

First we enter the x and y coordinates of the points and their masses.

```
> xp:=[-1, 4, 10];
```

```
xp := [-1, 4, 10]
```

(3.1.1.1)

```
> yp:=[3, 8, -5];
```

```
yp := [3, 8, -5]
```

(3.1.1.2)

```
> m:=[5, 7, 10];
```

```
m := [5, 7, 10]
```

(3.1.1.3)

Compute the mass.

```
> Mass:=Sum(m[i], i=1..3);
Mass:=value(Mass);
```

$$Mass := \sum_{i=1}^3 [5, 7, 10]_i$$

```
Mass := 22
```

(3.1.1.4)

Compute the moment about the y axis.

```
> M[y]:=Sum(xp[i]*m[i], i=1..3);
M[y]:=value(M[y]);
```

$$M_y := \sum_{i=1}^3 [-1, 4, 10]_i [5, 7, 10]_i$$

$$M_y := 123$$

(3.1.1.5)

Compute the moment about the x axis.

```
> M[x] := Sum(y[i]*m[i], i=1..3);  
M[x] := value(M[x]);
```

$$M_x := \sum_{i=1}^3 [3, 8, -5]_i [5, 7, 10]_i$$

$$M_x := 21$$

(3.1.1.6)

Compute the center of gravity.

```
> cog := [M[y]/Mass, M[x]/Mass];
```

$$cog := \left[\frac{123}{22}, \frac{21}{22} \right]$$

(3.1.1.7)

```
>
```