STAT101 Worksheet: Confidence Intervals

Key

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Identify the "given" information and determine the appropriate formula for the following situations. Calculate the confidence interval for one item representing each of the formulas.

$$\bar{x} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \qquad \bar{x} \pm Z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) \qquad \bar{x} \pm t_{\alpha/2,df}\left(\frac{s}{\sqrt{n}}\right) \qquad \hat{p} \pm Z_{\alpha/2}\left(\sqrt{\frac{\hat{p}\hat{q}}{n}}\right) \qquad n = \left(\frac{Z_{\alpha/2}\sigma}{E}\right)^2$$

1) On day two of a study on body temperatures, 106 temperatures were taken. Suppose that we only have the first 10 temperatures to work with. The mean and standard deviation of these 10 scores were 98.44°F and 0.30°F, respectively. Construct a 95% confidence interval for the mean of all body temperatures.

Given: $N = 106$, $n = 10$, $\bar{x} = 98.44\,°F$, $s = .30\,°F$, $t = 2.262$, 95% CL

$\bar{x} \pm t_{\alpha/2,df}\left(\frac{s}{\sqrt{n}}\right) \Rightarrow$ 95% CI $= 98.44 \pm 2.262\left(\frac{.30}{\sqrt{10}}\right) = 98.44 \pm .215\,°F = (98.225, 98.655)\,°F$

2) In a time use study 20 randomly selected managers were found to spend a mean time of 2.4 hours per day on paperwork. The standard deviation of the 20 scores was 1.30 hours. Construct a 98% confidence interval for the mean time spent on paperwork by all managers.

Given: $n = 20$, $s = 1.3$ hr., $t = 2.539$, $\bar{x} = 2.4$ hr., 98% CL

$\bar{x} \pm t_{\alpha/2,df}\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 2.4 \pm 2.539\left(\frac{1.3}{\sqrt{10}}\right) \Rightarrow$ 98% CI $= 2.4 \pm .738$ hr. $= (1.662, 3.138)$ hr.

3) A random sample of 19 women results in a mean height of 63.85 inches with a standard deviation of 2.5 inches. Other studies have shown that women's heights are normally distributed. Construct a 90% confidence interval for the mean height of all women.

Given: $n = 19$, $\bar{x} = 63.85''$, $s = 2.5''$, 90% CL, $df = 18$, $t = 1.734$

$\bar{x} \pm t_{\alpha/2,df}\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 63.85 \pm 1.734\left(\frac{2.5}{\sqrt{19}}\right) \Rightarrow$ 90% CI $= 63.85 \pm .995$ inches

$63.85 \pm 1.734(.574)$

$(62.855, 64.845)$ inches

4) The National Center for Education Statistics surveyed 4400 college graduates about the lengths of time required to earn their bachelor's degrees. The mean was 5.15 years and the standard deviation was 1.68 years. Based on the above information, construct a 98% confidence interval for the mean time required to earn a bachelor's degree by all college students.

Given: $n = 4400$, $s = 1.68$ yr, $\bar{x} = 5.15$ yr, 98% CL, $\alpha = .02$, $\alpha/2 = .01$, $Z_{\alpha/2} = 2.33$

$\xrightarrow{.9900}$ .01, 2.33

$\bar{x} \pm Z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 5.15 \pm 2.33\left(\frac{1.68}{\sqrt{4400}}\right) \Rightarrow$ 98% CI $= 5.15 \pm .059$ yr.

$5.15 \pm 2.33(.0253)$

$(5.071, 5.209)$ yr

5) A random sample of 60 female members of health clubs in Los Angles showed that they spend on average 4 hours per week doing physical exercise with a standard deviation of .75 hours. Find a 95% confidence interval for the population mean μ.

Given: $n = 60$, $s = .75$ hr., $\bar{x} = 4.0$ hr., $\alpha = .05$, $\alpha/2 = .025$, $Z_{\alpha/2} = 1.96$

$\xrightarrow{.9750}$ .025, 1.96

$\bar{x} \pm Z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 4.0 \pm 1.96\left(\frac{.75}{\sqrt{60}}\right) \Rightarrow$ 95% CI $= 4.0 \pm .19$ hrs $= (3.81, 4.13)$ hrs

6) A random sample of 20 married women showed that the mean time spent on housework by them was 29.8 hours a week with a standard deviation of 6.7 hours. Find a 95% confidence interval for the mean time spent on housework per week by all married women.

Given: $n = 20$, $s = 6.7$ hr., $\bar{x} = 29.8$ hr, $df = 19$, $t = 2.093$, 95% CL

$\bar{x} \pm t\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 29.8 \pm 2.093\left(\frac{6.7}{\sqrt{20}}\right) \Rightarrow$ 95% CI $= 29.8 \pm 3.14$ hrs. $= (26.66, 32.94)$ hrs

7) A company has a fleet of 100 airplanes for which they want to estimate air time (time spent flying). A sample of 32 of these planes gave a mean air time of 49 hours (s = 14.9 hours). Construct a 90% confidence interval on the mean air time for this fleet.

Given: $N = 100$, $n = 32$, $\bar{x} = 49$ hr, $s = 14.9$ hr, 90% CI, $Z_{\alpha/2} = 1.645$

$\bar{x} \pm Z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 49 \pm 1.645\left(\frac{14.9}{\sqrt{32}}\right) \Rightarrow$ 90% CI $= 49 \pm 4.33$ hrs $= (44.66, 53.33)$ hrs

8) Automotive engineers are continually improving their products/ Suppose a new type of brake light has been developed by General Motors. As part off a product safety evaluation program General Motors' engineers wish to estimate the mean driver response time to the new brake light. Fifty drivers are selected at random and the response time (in seconds) for each driver is recorded, yielding the following results: $\bar{x} = .72$ and s = .022. Construct a 99% confidence interval for the mean response time.

Given: $n = 50$, $\bar{x} = .72$, $s = .022$, 99% CL, $\alpha = .01$, $\alpha/2 = .005$, $Z_{\alpha/2} = 2.575$

$\bar{x} \pm Z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) \Rightarrow .72 \pm 2.575\left(\frac{.022}{\sqrt{50}}\right)$

99% CI $= .72 \pm .008$ sec $= (.712, .728)$ sec

9) A random sample of 45 life insurance policy holders showed that the average premiums paid on their life insurance policies was \$340 per year with a standard deviation of \$62. Construct a 90% confidence interval for the population mean.

Given: $n=45$, $\bar{x}=\$340$, $s=\$62$, 90% CL, $\alpha=.10$, $\alpha/2=.05$, $Z_{\alpha/2}=1.645$

$\bar{x} \pm Z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 340 \pm 1.645\left(\frac{63}{\sqrt{45}}\right) \Rightarrow$ 90% CI = \$340 ± \$15.20 = (\$324.80, \$355.20)

10) A process has been developed that can transform ordinary iron into a kind of super iron called *metallic glass*. Metallic glass is three to four times stronger than the toughest steel alloys. To estimate the mean temperature, μ, at which a particular type of metallic glass becomes brittle, 25 pieces of this metallic glass were randomly sampled from a recent production run. Each piece was subjected to higher and higher temperatures until it became brittle. The temperature at which brittleness first appeared was recorded for each piece in the sample. The following results were obtained: $\bar{x}$ = 480°F and s = 11°F. Construct a 95% confidence interval to estimate μ.

Given: $n=25$, $\bar{x}=480$ °F, $s=11$ °F, 95% CL, $df=24$, $t=2.064$

$\bar{x} \pm t_{\alpha/2, df}\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 480 \pm 2.064\left(\frac{11}{\sqrt{25}}\right) \Rightarrow$ 95% CI = 480 ± 4.54 °F = (475.46, 484.54) °F

11) Health insurers and the federal government are both putting pressure on hospitals to shorten the average length of stay (LOS) of their patients. A random sample of 27 hospitals in one state had a mean LOS in 1998 of 3.8 days and a standard deviation of 1.2 days. Construct a 98% confidence interval to estimate the population mean of the LOS for the state's hospitals in 1998.

Given: $n=27$, $\bar{x}=3.8$ days, $s=1.2$ days, $df=26$, 98% CL, $t=2.479$, $t_{\alpha/2, df}=t_{.01,26}$

$\bar{x} \pm t\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 3.8 \pm 2.479\left(\frac{1.2}{\sqrt{27}}\right) \Rightarrow$ 98% CI = 3.8 ± .573 days = (3.227, 4.373) days

12) A random sample of 50, 8 ounce cups of black "Early Riser" coffee dispensed by a new machine gave a mean of 11.0 mg. of caffeine (s = 7.1 mg). Construct a 90% confidence interval for μ, the mean caffeine content for cups dispensed by this machine.

Given: $n=50$, $\bar{x}=11$ mg, $s=7.1$ mg, 90% CL, $\alpha=.10$, $\alpha/2=.05$, $Z_{\alpha/2}=1.645$

$\bar{x} \pm Z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 11 \pm 1.645\left(\frac{7.1}{\sqrt{50}}\right) \Rightarrow$ 90% CI = 11 ± 1.63 mg = (9.37, 12.63) mg

13) The U.S Bureau of the Census conducted a survey of 5000 people and found that the mean income for a person with a bachelor's degree was \$38,973. The standard deviation in income for a person with a bachelor's degree was \$6,340. Construct a 98% confidence interval for μ, the mean income nationwide for persons with a bachelor's degree.

Given: $n=5000$, $\bar{x}=\$38973$, $s=\$6,340$, 98% CL, $\alpha=.02$, $\alpha/2=.01$, $Z_{\alpha/2}=2.33$

$\bar{x} \pm Z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) \Rightarrow$ 90% CI = \$38,973 ± \$208.91 = (\$38,764.09, \$39,131.91)

14) In a Roper poll of 3000 working men, 56% said "they feel guilty that they don't spend more time with their families." Construct a 98% confidence interval for the proportion of all working men who hold this view.

Given: $\hat{p}=.56$, $\hat{q}=.44$, $\alpha=.02$, $\alpha/2=.01$, $Z_{\alpha/2}=2.33$, 98% CL, $n=3000$, $f=1680$

$\hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .56 \pm 2.33\sqrt{\frac{(.56)(.44)}{3000}} \Rightarrow$ 98% CI = .56 ± .02 = (.54, .58)

Where f come from? $\hat{p}=\frac{f}{n}$, $.56=\frac{f}{3000}$, $f=1680$

15) A bank took a sample of 100 of its delinquent credit card accounts and found that the mean owed on these accounts was \$2,130 with a standard deviation for delinquent credit card accounts at \$578. Give a 97% confidence interval for the mean amount owed on all delinquent credit card accounts for this bank.

Given: $n=100$, $\bar{x}=\$2130$, $s=\$578$, 97% CL, $\alpha=.03$, $\alpha/2=.015$, $Z_{\alpha/2}=2.17$

$\bar{x} \pm Z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 2130 \pm 2.17\left(\frac{578}{\sqrt{100}}\right) \Rightarrow$ 97% CI = \$2130 ± \$125.43 = (\$2004.57, \$2255.43)

16) A random sample of 100 movie theaters showed that the mean price of a movie was \$7.00 with a standard deviation of \$.80. Construct a 99% confidence interval for the population mean μ.

Given: $n=100$, $\bar{x}=\$7$, $s=\$.80$, 99% CL, $\alpha=.01$, $\alpha/2=.005$, $Z_{\alpha/2}=2.575$

$\bar{x} \pm Z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 7 \pm 2.575\left(\frac{.8}{\sqrt{100}}\right) \Rightarrow$ 99% CI = \$7 ± \$.21 = (\$6.79, \$7.21)

17) In a *Time/CNN* telephone poll of 1012 adult Americans, 11% of the respondents said that Ronald Regan was a great president. Give a 98% confidence interval for the proportion of all adult Americans who think that Regan was a great president.

Given: $n=1012$, $\hat{p}=.11$, $\hat{q}=.89$, 98% CL

$\hat{p}=\frac{f}{n}$, $.11=\frac{f}{1012}$, $f=111$

$\hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .11 \pm 2.33\sqrt{\frac{(.11)(.89)}{1012}} \Rightarrow$ 98% CI = .11 ± .023 = (.097, .133)

18) Find n: A researcher wants to determine the 99% confidence interval for the mean number if hours per week that adults spend doing community service. How large of a sample should the researcher select so that the estimate will be within 1 hour of the population mean? Assume that the standard deviation for hours spent per week by adults doing community service is 3.

$n=\left(\frac{Z_{\alpha/2}\cdot\sigma}{E}\right)^2 \Rightarrow n=\left(\frac{2.575(3)}{1}\right)^2 \Rightarrow n=59.68 \Rightarrow 60$ round up