Intermediate Laboratory Manual

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The Photoelectric Effect

At virtually the same time (1897) Heinrich Hertz discovered the photoelectric effect while investigating electromagnetic waves and J. J. Thompson discovered the electron. Hertz found that when light was shown upon a metal surface in an evacuated chamber a current flowed in an external circuit.
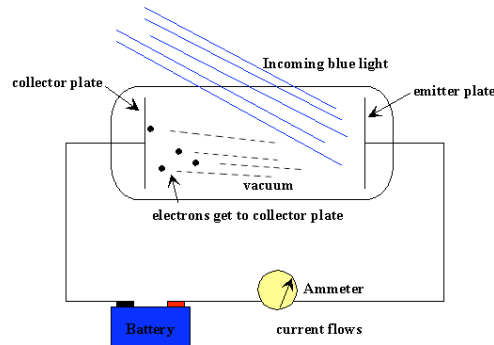

Figure 1

As illustrated in the figure incoming light strikes a metal plate ejecting electrons. The electrons cause a current flow in the external circuit where the current is read on the ammeter. The battery is used to bias the circuit and as will be seen to determine the stopping voltage.

Attempts to explain the photoelectric effect using classical theory, viz., Maxwell's equations, were not successful. The classical point of view held that light was a wave phenomenon and therefore photoelectrons will be emitted after enough energy is extracted from the incident wave; in other words the ejection of photoelectrons should be a function of the wave intensity and for lower intensities it would take longer for the photoelectrons to be emitted. In 1902 Lenard did a series of experiments investigating the photoelectrons versus light intensity and discovered that if the light was not of a certain threshold frequency photoelectrons were not emitted, which was also contrary to the classical point of view. He also observed that there was no delay time for light with low intensity as expected, rather once the threshold frequency was reached photoemission was immediate. The classical wave theory simply was not able to explain the photoelectric effect.

In 1905 Einstein proposed that light behaved as a particle in the sense that a single "photon" of light carried an energy that was proportional to its frequency. When light was shone on a metal surface a photon could interact with an electron at or near the surface and impart all of its energy to the electron. If the energy imparted was sufficient to overcome the binding energy between the electron and the surface the electron will be ejected and the difference between the energy required to dislodge the electron and energy imparted by the photon will manifest as the ejected electron's kinetic energy. The energy required to dislodge an electron is referred to as the work function and is analogous to the ionization energy associated with an element, for example the energy required to ionize hydrogen is 13.6 eV, which incidentally is far more energy than is required to eject photo electrons. The description may be expressed symbolically as:

$$T_{electron} = h\nu - \phi \qquad (1)$$

T is the kinetic energy of the electron, hν is the incident energy of the photon where h is Plank's constant (h = 6.626 x 10^{-34} J-sec) and ν is the frequency of the photon. If this theory is correct it immediately explains why the intensity of light below a certain threshold caused no photo-electrons to be created. Suppose hν is less than the work function so that T is less than zero; however this is impossible as the electron's velocity

would be imaginary. Put differently if h**ν** is not greater than the work function there is not enough energy in the incoming photon to eject an electron.

The assumption that a photon imparts all of its energy to the electron upon interaction also resolves the issue associated with the time required for a photoelectron to be ejected by the energy extracted from the wave fronts since the photon-electron interaction is immediate. As the photon beam is increased in intensity the number of photoelectrons emitted will increase but there will be no noticeable time delay as the intensity is decreased. However, no photoelectrons will be emitted if the incident photon do not have an energy greater that the threshold energy.

If light of a particular frequency is shown on a metal plate then the kinetic energy is given by equation 1. In general the velocities of the emitted electrons are randomly oriented; however some will have velocities pointing directly at the collector plate in figure 1. Now if an electric field is created between the collector and the emitter plate by the voltage source shown it will either attract or repel the incident electrons depending upon its polarity. Assume the battery is adjusted to repel the incident electrons; at some voltage, V_s, the electrons will not have sufficient kinetic energy to reach the collector plate and current will cease causing the ammeter to read zero. Note – the random orientation of the velocity distribution of the ejected photoelectrons means that as the stopping voltage, as it is called, is increased the current will diminish. The stopping voltage multiplied by the electron's charge is equal to the maximum kinetic energy of the incident photoelectrons. Therefore:

$$T_{max} = eV_s = h\nu - \phi$$

$$V_s = \left(\frac{h}{e}\right)\nu - \frac{\phi}{e} \qquad (2)$$

Experimentally light of know frequencies will be shone upon a metallic plate and the corresponding stopping voltage will be determined. A plot of V_s versus frequency will then be made. The data should be plotted using a spreadsheet and the best fit obtained. Assume the electron charge is known (e = 1.6021 x 10^{-19} C) and determine h and ϕ. Can you identify the material from which the photoelectrons were obtained using the data in figure 2?

Element	eV	Element	eV	Element	eV	Element	eV	Element	eV	Element	eV
Ag:	4.52-4.74	Al:	4.06-4.26	As:	3.75	Au:	5.1-5.47	B:	~4.45		
Ba:	2.52-2.7	Be:	4.98	Bi:	4.34	C:	~5	Ca:	2.87		
Cd:	4.08	Ce:	2.9	Co:	5	Cr:	4.5	Cs:	2.14		
Cu:	4.53-5.10	Eu:	2.5	Fe:	4.67-4.81	Ga:	4.32	Gd:	2.9		
Hf:	3.9	Hg:	4.475	In:	4.09	Ir:	5.00-5.67	K:	2.29		
La:	3.5	Li:	2.93	Lu:	~3.3	Mg:	3.66	Mn:	4.1		
Mo:	4.36-4.95	Na:	2.36	Nb:	3.95-4.87	Nd:	3.2	Ni:	5.04-5.35		
Os:	5.93	Pb:	4.25	Pd:	5.22-5.6	Pt:	5.12-5.93	Rb:	2.261		
Re:	4.72	Rh:	4.98	Ru:	4.71	Sb:	4.55-4.7	Sc:	3.5		
Se:	5.9	Si:	4.60-4.85	Sm:	2.7	Sn:	4.42	Sr:	~2.59		
Ta:	4.00-4.80	Tb:	3	Te:	4.95	Th:	3.4	Ti:	4.33		
Tl:	~3.84	U:	3.63-3.90	V:	4.3	W:	4.32-5.22	Y:	3.1		
Zn:	3.63-4.9	Zr:	4.05								

Figure 2

Experimental Procedure

Familiarize yourself with the photoelectric apparatus; the controls of particular importance are the voltage adjustment control, the light intensity control, the voltage polarity switch, and the current voltage switch. You will notice that the light source can be moved to various positions and you will take data from three positions that are set roughly 5 cm apart. Position the light source at about mid position, say 25 cm, and then adjust the intensity to a moderate level. Place the red filter, 635 nm, in the filter holder and put the current-voltage switch in the current position. Begin to adjust the voltage control until the current is zero. (You can adjust the sensitivity of the device with the current multiplier control but should obtain your final stopping voltage with the current multiplier in its most sensitive position.) Once you have obtained a null current switch the current-voltage control to the voltage position to determine the stopping voltage. Repeat the procedure for each of the available filters and record the stopping voltage and the wave length in a table like the one shown in figure 3.

Record Position of Lamp: _____ cm.

Filter Color	Filter Wave Length in nm	Stopping Voltage
Red	635	.35
Yellow I	570	
Yellow II	540	
Green	500	
Blue	460	

Plot you data and find the best fit for each of the data sets. Make a plot of V versus frequency and interpret the meaning of the slope and intercept. What is your best estimate of h, Planks constant? In your report show how you did the calculation and explain your error analysis.

Questions that should be addressed in your lab report:

1. In your plot of stopping voltage vs. frequency what is the significance of the slope? The y intercept? The x intercept? Explain each in as much detail as you can – this does not mean your answer needs to be lengthy but should be concise.

2. Estimate the error associated with your data. Use the three graphs to estimate the minimum, maximum, and best value of h obtained from your experimental data. Discuss both qualitatively and quantitatively.

Millikan's Oil Drop Experiment

The Millikan oil drop experiment allows one to determine the charge of an electron by observing the motion of oil drops that have been passed through an atomizer; in the process of being atomized the tiny oil drops acquire a charge and this excess charge allows one to manipulate the motion of the oil drops by application of a uniform electric field that is created between two metal plates. This ability to manipulate the motion allows one to derive an equation that can be used to determine the charge on an electron. The experiment not only allowed a determination of the electronic charge it also clearly demonstrated that charge on the electron is quantized, as will become clear as the details of the Millikan experiment are examined.

Background

Consider an oil drop that is introduced between the plates of a parallel plate capacitor. Recall that if a potential difference of V volts is impressed across the plates the field strength is E = V/d, where d is the separation of the plates. If the plates are closely spaced and fairly large the field may be considered uniform in the center region. Assume the plates are initially short circuited so the field is zero and that the motion of the oil drop can be observed. The forces acting on the oil drop are mg, the force of gravity, and a drag force that will be assumed is proportional to the velocity of the particle. Applying Newton's second law to the situation gives the equation of motion:

$$F = ma = bv - mg \qquad (1)$$

The constant b is associated with the drag force on the particle. If the oil drop begins to fall its velocity will increase as will the drag force b since it depends on velocity. Eventually the gravitational force and the drag force will become equal and the net force of the oil drop will achieve a terminal velocity given as:

$$0 = bv_f - mg \ \ or \ \ v_f = \frac{mg}{b} \qquad (2)$$

Next consider the same situation but with the electric field turned on causing the particle to move upward; in this case the drag force reverses direction, and the terminal velocity is reached very quickly.

$$Eq - mg - bv_r = 0 \qquad (3)$$

Equations (2) and (3) are now solved simultaneously eliminating b between them to arrive at:

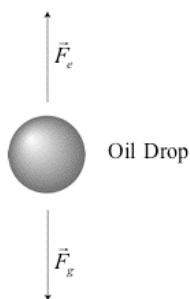$$q = \frac{mg(v_f + v_r)}{Ev_f} \qquad (4)$$

Figure 1

In figure 1 the oil drop is shown under the influence of the electric force and gravity, which would be the condition if the forces were equal and opposite so that the drag force, which is proportional to v, is zero. As a good exercise draw the cases when the droplet is moving upward and downward so that you understand the directions of all the forces when the net force on the droplet is unbalanced.

All of the quantities in equation 4 can be measured directly save the mass of the droplet. The mass of the droplet may be written in terms of the density of the oil as:

$$m = \rho V = \frac{4}{3}\pi a^3 \rho \quad (5)$$

Equation 5 assumes the droplet is a sphere of radius a. While the density of the oil can, of course, be determined by direct measurement, the radius of the drop will vary. Fortunately the radius of the oil drop is related to its terminal velocity, v_f through Stoke's Law. This last statement may seem as though it came out of left field, although a moment's reflection should make it plausible. The drag force originated because the drop is moving through a viscous medium and it makes sense to assume that it should be proportional to the cross sectional area. The expression from Stoke's law states that:

$$a = \sqrt{\frac{9v_f \eta}{2g\rho}} \quad (6)$$

So it looks like we can use the apparatus to directly measure the velocities when the drop is rising and falling, allowing the voltage and thus E to be determined, where a can be obtained from equation (6). In the experiment the oil drop is watched for a long time moving up and down. The distances and times are recorded to obtain values of the velocities as the particle rises and falls. When the experiment was being done by Millikan a single drop would be monitored for several hours at a time, which allowed very accurate determination of the velocities in equation 4.

Well unfortunately our work is not quite finished. Equation (6) is only good for velocities greater than .001 m/sec and this condition will not be satisfied in our experiment. It is necessary to add a correction factor to equation (6) to make things work. The correction factor required is:

$$\eta \to \eta \left(\frac{1}{1 + \dfrac{b}{pa}} \right) \quad (7)$$

Where b is a know constant, and p is the barometric pressure. Inserting (7) the expression for a is:

$$a = \sqrt{\left(\frac{9v_f}{2g\rho} \right)\left(\frac{1}{1 + \dfrac{b}{pa}} \right)} \quad (8)$$

But alas there is yet another problem – we need an expression for a, but a is found on both sides of the equation. Square equation (8) and solve for a using the quadratic equation to obtain:

$$a = \frac{-b}{2p} + \sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9v_f \eta}{2g\rho}} \qquad (9)$$

Substituting a into equation 4 and using the fact that E = V/d the expression for q is:

$$q = \left[\frac{4}{3}\pi d\rho g\right]\left[\frac{-b}{2p} + \sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9v_f \eta}{2g\rho}}\right]^3 \left[\frac{v_v + v_r}{Vv_f}\right] \qquad (10)$$

The first term in the expression for q will only have to be calculated once for a given run, while the second must be calculated for each drop, and the third term will change each time the voltage is altered.

You should have some appreciation from the foregoing that when Millikan carried out his experiment a great deal of time and effort was devoted to making the measurements and performing the calculations. Millikan did not have a computer or even an electronic calculator at the ready. In the derivation above everything remained very straightforward until the expression for the mass was introduced requiring the use of Stoke's law and the correction factor.

In our experiment we will use small plastic spheres with a known diameter and density. This means the problems associated with oil drops of various radii and different masses will be eliminated and our analysis can be done with equations one through five; avoiding the necessity of dealing with all the complications. Had Millikan be able to secure these small plastic spheres his labors would have been significantly reduced.

Armed with knowledge that the mass and diameter of the spheres are constant great simplification occurs in the experimental procedure. Using the concepts developed in equations 1 through 5 above we shall develop the analysis that you will use in this experiment.

Using equation (1) the equation of motion for the sphere in free fall is:

$$v_f = \frac{mg}{b} \qquad (11)$$

If a voltage is impressed across the plates of the Millikan oil drop apparatus an electric field, E = V/d is created that will exert an upward force on the plastic sphere provided the upper plate is positive with respect to the lower plate. Assuming the voltage is large enough the sphere will move upward and the equation for the equilibrium velocity is:

$$qE - mg - bv_e = F = 0 \quad so \quad q = \frac{mg + bv_e}{E} \qquad (12)$$

Note that equations 11 and 12 have different velocities with subscripts reminding you that in equation 11 the f stands for free velocity and in 12 the e reminds you it is the velocity resulting due to the electric field. Since we are using small plastic spheres with known diameter and density a value for m may be obtained, which allows us to use equation (11) to estimate b. Therefore using (11) and (12) together the expression for q may be written as:

$$q = \frac{b(v_f + v_e)}{E} = \frac{bd(v_f + v_e)}{V} \quad and \quad \Delta q = \frac{bd}{V}(v_e - v_e') = \frac{bd}{V}\Delta v_e \qquad (13)$$

By making measurements of the velocities and knowing b equation (13) can be used to calculate q and/or δq. Using the first equation the free fall velocity and the velocity when E is impressed are required. These values would be determined at a given voltage; shorting the plates (switch in middle position) puts the sphere in free fall and you can measure the time it takes the sphere to pass through 2 divisions or 1 mm. Placing the switch in the up position allows you measure the velocity under the influence of the field to obtain v_e. The advantage of the direct method is that you make measurements on one particle at a time at the same voltage setting. Alternately you can measure v_e for particles at two voltage settings to obtain v_e and v_e' to use the equation for δq.

An alternate method is to adjust the voltage until the force exerted by the electric filed just cancels the force of gravity on the sphere. Then

$$qE = mg \quad and \quad v_e = 0$$

$$q = \frac{mg}{E} = \frac{mgd}{V} \qquad (14)$$

Because the spheres are of a known diameter and density m is calculated directly. For example if the sphere have a diameter of 1.01 microns and a density of 1.05 gm/cm³ the mass is calculated as:

$$m = \rho V = 1050 \frac{kg}{m^3} \frac{4}{3} \pi \left(.505 \times 10^{-6}\right)^3 m^3 = 5.6 \times 10^{-16} kg \quad (15)$$

Since the spacing between the plates is 4 mm and g = 9.8 m/sec² the numerator in (14) may be calculated for all calculations. The result is:

$$mgd = 5.6 \times 10^{-16} kg \times 9.8 \frac{m}{s^2} \times 4.0 \times 10^{-3} m = 2.2 \times 10^{-17} Nm \quad (16)$$

For a given voltage the value of q can now be calculated. This is by far the easiest method to calculate q. You will obtain various values of q for different voltages because the spheres will contain 1, 2, or more electrons. The more electrons a given sphere has the lower the voltage required to hold a particle stationary.

<div align="center">Experimental Procedure 🍎</div>

Set up the Millikan apparatus connecting a 6.3-volt supply to the lamp and the high voltage supply to the appropriate terminals. Be sure to observe the proper polarity. Place the switch in the middle position or shorted position.

Remove the top cover of the chamber and place a small piece of paper in the center of the chamber with the lamp illuminated. You should be able to see a bright spot on the paper when in the center of the chamber if not adjust the lamp until you do. Next reassemble the chamber and push the tube that delivers the atomized spheres to the chamber fully in. Adjust the microscope until you can focus the edge of the tube. Note – you may find that the tube is not long enough to be within the range of vision of the microscope in which case you will have to insert a thin wire through the tube and focus on the wire. The apparatus is now ready for viewing the droplets.

The spheres will be provided to you in a solution of distilled water and isopropyl alcohol. The spheres come in a concentrated solution and are diluted with approximately 1 ml of distilled water and ½ ml of alcohol per fifteen drops of concentrated sphere solution. Place a small amount of solution in the atomizer.

Squeeze the atomizer and look thought the telescope. If you do not see the spheres adjust the microscope slightly. The focal plane is very sharp and you may have to do some fine-tuning until the spheres are clearly visible. If you can not see them after some trial and error ask for assistance.

With the switch in the middle position, capacitor plates shorted and thus no field, you should see some of the spheres drifting upward under the influence of gravity. (Note the telescope inverts the image so the spheres drifting upward are actually being influence by gravity and when a sufficiently large field is applied the spheres will drift downward; in other words up is down and down is up. Welcome to Alice's world. If you look very hard you may even see the Cheshire Cat.)

The Cheshire Cat

Estimate b

Focus on one sphere at a time and record the time for the drop to traverse two lines on the telescope reticle, which is a distance of 1 mm. Record the times for ten drops and compute the average velocity v_f. Use this value along with the mass of the drop to determine a value for b using equation (11). (Note the telescope has a magnification 20 X and each division on the scale is .5 mm)
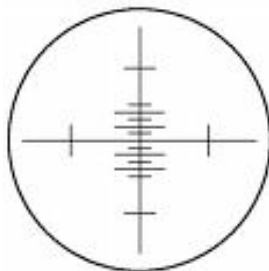

Figure 2 – Example Reticle

The lines on the telescope in the apparatus go across the entire field of vision. As an example calculation suppose an experimenter finds an average value of $v_f = 4.7 \times 10^{-5}$ m/sec, the b = $mg/v_f = 1.2 \times 10^{-13}$kg/s. Note – your values may be very different from the example. If you record ten values for v_f they should all be roughly the same. If you have values that are very different what might be the cause? To determine your best value for v_f find the velocities of ten spheres and calculate the best value of v_f and its standard deviation. Estimate the error in b. This means you will have to find the error in m and the error in v_f. (Assume g = 9.81). Your final answer should be expressed as:

$$b = b_{avg} \pm \delta b$$

First Determination of q

In the first exercise you should have developed a pretty good technique to determine the free fall velocity of the drops. In this part take data for 5 drops. Select a voltage to apply to the capacitor plates. Pick a value

between 100 and 150 volts. Inject some drops and first determine v_f for a drop. Once you have done that find v_e for the same drop. Use equation (13) to find q.

<div align="center">Determination of q – Method 2</div>

In this part of the experiment balance the force of gravity with the electric field and use equation (14) to determine q. You should be able to find several voltages that will balance the gravitational force. A voltage in the neighborhood of 140 volts should work for spheres with one excess electron and about half that for two and so on. See if you can determine equilibrium voltages for 15 spheres. The best way to do this is to inject some particles and focus on a particular sphere. Turn on the voltage and adjust the voltage until you can hold a sphere in equilibrium. By starting at various initial voltages you should get several different voltages that will put a sphere in equilibrium.

Imagine for a moment that you have just determined the charge on an electron using the ingenious methods of Millikan. Now how can you determine the mass of the electron? The answer: do a second experiment in which you can determine the charge to mass ratio for an electron, then if you know e (the electronic charge) and the ratio of e/m then it follows that the mass of the electron can be determined. This is precisely what we are going to do; assume that we know e, find e/m, and then determine our best estimate of the electron mass.

The method is elegant in its simplicity. Since an electron carries a charge it can acquire kinetic energy by being accelerated in a uniform electric field. If an electron "falls" through a potential difference V it acquires kinetic energy that is given as eV or:

$$eV = \frac{1}{2}mv^2$$

If the electron is injected into a region in which there is a uniform magnetic field that is perpendicular to the electron's velocity its trajectory will be circular as it will be deflected by the magnetic force, and hence

$$F = evB = \frac{mv^2}{r} \ \ so \ \ eB = \frac{mv}{r}$$

using the first equation v may be eliminated in the second and after some rearrangement the ratio of e/m may be written as:

$$\frac{e}{m} = \frac{2V}{r^2 B^2}$$

The experimental quantities to be determined are V, the accelerating voltage, r the radius of the electron orbit, and B the strength of the magnetic field.

The experimental set up consists of an evacuated tube in which is located a filament. The filament is the electron source; the electrons are generated by passing a current through the filament (usually made of tungsten or an alloy of tungsten) heating it, which literally boils electrons off the surface of the filament. A voltage is then impressed between the filament and a plate with a slit in it producing an electron beam. By varying the voltage the injection velocity can be changed. The voltage and thus the injection velocity of the electrons is one of the experimental parameters that will be of interest in the experiment. The radius of the electron orbit is measured directly using the scale that is behind the tube.
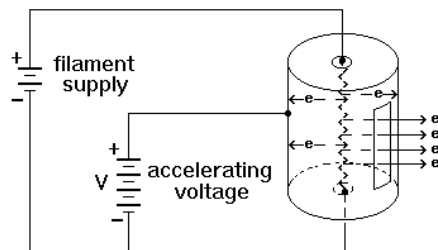

Figure 1 – Simplified Electron Source

Figure 1 is a simplified version of an electron gun. The wavy line connected to the filament voltage is the electron source and the cylinder surrounding the filament is the accelerating anode. The anode supply

provides the accelerating voltage for the electrons and most are absorbed by the anode, however those forming the electron beam pass through the vertical slit forming the beam. The electron gun is located in the center of the e/m tube so that they are injected into a region in which there is a uniform magnetic field.

The tube containing the electron source contains neon gas and when the electrons collide with the neon atoms they are excited and give off a bleu-violet light that makes the electron's path visible. As mentioned above there is a scale located behind the tube that allows you to determine the radius of the orbit.

The magnetic field is provided by a Helmholtz coil, which consists of two coils arranged as shown in figure 2:
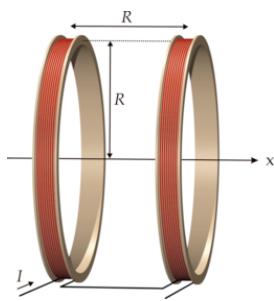

Figure 2

Notice that the coils have a radius R and are separated by the same distance, R. The electron source is located along the x-axis and is at the center. The Helmholtz coils produces a uniform[1] magnetic field in this region and the strength of the filed may be calculated using the Biot-Savart equation. The result of the calculation is:

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 NI}{R} = \frac{8\mu_0 NI}{5\sqrt{5}R}$$

Where N is the number of turns in the coil, I is the current flowing in the coils, and R is the radius of the each coil. The equation for B may be written as a constant, k, times I so let $B = kI$ and then the resulting equation for e/m may be written as:

$$\frac{e}{m} = \frac{2V}{r^2 k^2 I^2}$$

There are several ways to determine the charge to mass ratio, however in this experiment you will select an accelerating voltage and measure r, the radius of the electron orbit as a function of I. For a given value of V the equation above may be recast as:

$$\frac{\alpha}{r^2} = \left(\frac{e}{m}\right) I^2 \quad \alpha = \frac{2V}{k^2}$$

Plot $1/r^2$ versus I^2 and the resulting slope is the ratio of e/m. Select several different value of V and for each measure the radius of the electron orbit as a function of current.

The accepted value for the charge-to-mass ration of the electron is $e/m = 1.7588196 \times 10^{11}$ C/kg.

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[1] Uniform is a relative term. The field at the center is most uniform for the geometry shown. The calculation of the field and some comments regarding uniformity are discussed in the appendix.

Experimental Procedure – Additional Details

The experimental system consists of a partially evacuated tube containing neon gas at low pressure and an electron source. There are three power supplies; a low voltage supply that operates the Helmholtz coils providing the magnetic field and a second supply that provides filament voltage to the electron source and the accelerating potential for the electron gun. Take care not to exceed the voltage/current limits for the Helmholtz coil. Power up the apparatus as follows:

1. Locate the high voltage control and turn it fully ccw. Turn on the tube power supply – the switches on the supply are clearly marked as to function and it is your responsibility to read the legends on the apparatus before throwing switches. If you have any question as to what to do ask before making a potentially costly error. You should see the filament begin to glow a reddish orange color.

2. Gradually advance the high voltage control until you can see the path of the electron beam as it ionizes the neon gas forming a blue-violet "vapor" trail. Select a voltage on the order of 200 – 225 volts. Once a voltage has been selected it will be maintained for the data run. You will collect data for at least 4 accelerating voltages. Keep anode voltage less than 400 V.

3. Turn on the Helmholtz supply and gradually increase the voltage control until you see a current indication on the current meter. (The Helmholtz supply has both a current meter and a voltmeter – the voltage is never to exceed 9 volts and the current should be kept below 2 amps.) Turn the voltage up until the arc of the electron beam is completely inside the tube and you can make the radius of arc small enough so that at least 5 measurement of the radius can be made without drawing more than 2 amps of current. If this is not possible increase the accelerating voltage until you are satisfied with your ability to control the radius in such a manner that at least 5 measurements of the radius can be made. Note the Helmholtz current is most easily controlled using the control on the external supply rather than the current adjustment shown in figure 3. Set the current adjustment shown in figure to mid range and control the current with the control on the supply.
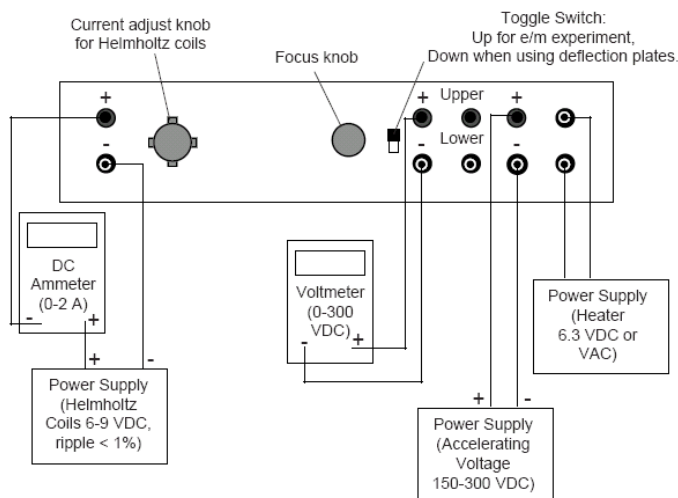

Figure 3 – Pasco Controller

4. Take data as above for four accelerating voltages. Record your data in a table such as the one shown in figure 2.

Accelerating Voltage_____	Current in Helmholtz Coil	Radius of Orbit
I_1/R_1		
I_2/R_2		
I_3/R_3		
I_4/R_4		
I_5/R_5		

Figure 4 - Sample Data Table

From your data plot $1/r^2$ versus I. What is the significance of the slope? You may find it convenient to rewrite the equation for $1/r^2$ vs. I in terms of all the individual quantities. This is left as an exercise and you may well want to spend a few moments thinking about putting your data on a spreadsheet and letting the spreadsheet do the number crunching. Ten minutes spent in organizing the spreadsheet will save you a great deal of grief in the long run.

Plot your data and determine the best value of e/m by fitting that data using a linear regression, which is another name for a least square fit.

Questions

1. Identify sources of error and indicate which will contribute most to the total error.

2. An electron is shot into a region in which an electric field acts to deflect it upward. The electric field acts over a distance d and the electron travels an additional distance D before it collides with a phosphorous plate emitting a spot of light so that it defection distance from the horizontal injection line can be determined. A variable magnetic field is turned on and is adjusted so that the deflection caused by the electric field is just cancelled. (a) Draw an accurate diagram of the situation described and show the directions of the E and B fields. (b) Show that by knowing E and B when the electron deflection is eliminated the injection velocity is v = E/B. (c) Find an expression for the charge to mass ratio of the electron.

The law of Biot-Savart may be used to calculate the magnetic field due to current flows. If the geometry is simple, as is the case for Helmholtz Coils, the calculation is fairly easy.
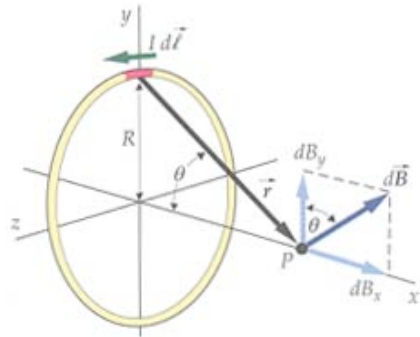

Figure 5

In the figure above a current flows in the loop of wire creating a differential element of magnetic field, dB, as shown. The horizontal components of the field, dB_y, cancel one another in pairs as the integration is carried out. The vertical components, dB_x, add to one another to create a field that is along the x-axis of the figure. The law of Biot-Savart is:

$$d\bar{B} = \frac{\mu_0}{4\pi} \frac{id\bar{l} \times \bar{r}}{r^3}$$

The differential element is a vector that is perpendicular to the plane of dl and r as shown in the figure. As should be clear as the integration is carried out only the components dB_x contribute to the field at the point P. It should also be clear that the angle between dl and r is $\pi/2$ so that dB_x is:

$$dB_x = \frac{\mu_0}{4\pi} \frac{id\bar{l} \times \bar{r}}{r^3} \sin\theta = \frac{\mu_0 i}{4\pi} \frac{Rd\phi}{r^2} \sin\theta = \frac{\mu_0 i}{4\pi} \frac{Rd\phi}{r^2} \frac{R}{\sqrt{R^2 + x^2}}$$

In this equation R is the radius of the Helmholtz coil and r is the distance from the differential element of current to the point P so that the expression above becomes:

$$B_x = \frac{\mu_0 i}{4\pi} \int_0^{2\pi} \frac{R^2 d\phi}{\left(R^2 + x^2\right)^{3/2}} = \frac{\mu_0 i}{4\pi} \frac{2\pi R^2}{\left(R^2 + x^2\right)^{3/2}}$$

Now because there are two coils each of which contributes the same field at x the result above must be doubled and evaluated at x = R/2.

$$B_x = \frac{\mu_0 i R^2}{\left(R^2 + x^2\right)^{3/2}} \bigg|_{x=R/2} = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 I}{R} \quad and \ for \ N \ turns \ B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 NI}{R}$$

Evaluating the constants the expression for the magnetic field may be put in the form:

$$B = \frac{32\pi NI}{5\sqrt{5}R} \times 10^{-7} \ tesla$$

Helmholtz coils are important because the field at "mid plane" is uniform and independent of the location on the mid plane. It is interesting to note that the uniformity is dependent on the geometry of the coils. The choice of coils with a radius of R and the spacing of the coils a distance equal to R is not an accident. Indeed as the distance between the coils varies the uniformity of the field is affected. For additional information follow the link: http://physicsx.pr.erau.edu/HelmholtzCoils/index.html

<div align="center">Note on Uniformity of the B field</div>

The expression above is for B at the exact center of the ring; the expression may be generalized as[2]:

$$B = NI\mu_0 \int_0^{2\pi} \frac{R(R - \cos\theta)d\theta}{\left(R^2 + b^2 + r^2 - 2Rr\cos\theta\right)^{3/2}}$$

This is the expression for B due to one of the coils where is the number of turns, R is the radius of the coil (in meters), b is the distance from the plane of the coil, and r is the distance from the axis of symmetry (perpendicular to the plane of the coil and passing through the center of the coil). The integral is often written with R = a, which gives it a little nicer appearance and is consistent with the link above. Unfortunately the integral above cannot be integrated by ordinary means, but is an elliptic integral of the second kind and is most readily integrated by numerical means.

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[2] The equation was taken from the link cited on this page: http://physicsx.pr.erau.edu/HelmholtzCoils/index.html

The era of "modern physics" began in the late nineteenth century perhaps most notably with the work of Plank in his attempt to understand black body radiation and concluding that the only way to explain the black body spectrum was to assume energy was quantized, which was a concept foreign to prevailing theory. Plank was so concerned about his iconoclastic conclusion that he hesitated for 17[3] years before publishing his work. In the early twentieth century Einstein (1905) used the concept of energy quantization to explain the photoelectric effect and Bohr (1913) used the concept in his astonishingly accurate Bohr model of the atom. The Bohr theory of the hydrogen atom may be thought of as the beginning of quantum mechanics and in many respects represents a bridge between classical theory and quantum theory. Indeed Bohr used classical notions and added the concept of quantization of momentum to develop his model of the hydrogen atom. We begin by looking at the problems facing physicists trying to explain the stability of the hydrogen atom. At the beginning of the twentieth century the existence of the electron was known as well as the fact that the bulk of an atoms mass was contained in the nucleus. Atoms were known to be stable and electrically neutral. Early attempts to employ classical electrodynamics to describe the situation failed and in fact suggested that atoms containing electrons in motion could not be stable; the electron orbiting the nucleus accelerates and according to classical theory will radiate energy and thus the electron orbit will decay and will fall into the nucleus. The existence of stable atoms was thus a problem; a problem beyond explanation using classical theory.

Bohr used a classical model in concert with some bold assumptions that flew in the face of classical theory but nonetheless explained with great accuracy the hydrogen spectrum. The Bohr model assumes the following:

Electrons are bound to the nucleus by the Coulomb force and occupy orbits for which angular momentum is quantized, which results in specific radius for different values of angular momentum. The angular momentum is quantized as $L = nh/2$.

Electrons in such orbits do not emit any radiation and are stable

If an electron moves from one orbit to another it emits radiation that is equal to the energy difference between the two states manifesting as a photon with energy $h\nu$.

For an electron orbiting about a nucleus that is assumed to be a fixed center of force the expression for the centripetal force keeping it in orbit is:

$$F = \frac{ke^2}{r^2} = \frac{mv^2}{r}$$

and the total energy of the electron is E=T+U or using the above equation and the expression for the electric potential energy of the electron E may be written as:

$$E = T + U = \frac{1}{2}mv^2 - \frac{ke^2}{r} = \frac{ke^2}{2r} - \frac{ke^2}{r} = -\frac{ke^2}{2r}$$

The expression for the total energy is inversely proportional to r and for each energy there corresponds a particular radius. If the angular momentum is quantized this means that $L = nh/2\pi$, which is one of the Bohr assumptions stated above. So

---

[3] This is a recollection and the actual time may be different. I have made an effort to research this fact but have not yet found a definitive confirmation.

$$L = mvr = nh/2\pi \quad and \quad v = \sqrt{\frac{ke^2}{mr}}$$

$$L = mvr = m\sqrt{\frac{ke^2}{mr}}r = \sqrt{mrke^2} = \frac{nh}{2\pi}$$

From the last equation the radius of an electron for a given angular momentum may be written as:

$$r_n = \frac{n^2h^2}{4m\pi^2ke^2}$$

This value for r_n may be substituted into the expression for E to find the energy corresponding to a give value of n, the smallest radius (n=1) possible, is called the Bohr radius and has a value of 0.529 x 10^{-10} m. The energy is given as:

$$E_n = -\frac{4m\pi^2k^2e^4}{2n^2h^2} \quad and \quad r_n = \frac{n^2h^2}{4m\pi^2ke^2} \quad r_1 = 0.529 \times 10^{-10} m$$

For a transition from one energy level to another the energy difference is given as:

$$\Delta E_{mn} = E_m - E_n = -\frac{4m\pi^2k^2e^4}{2m^2h^2} - \left(-\frac{4m\pi^2k^2e^4}{2n^2h^2}\right) = \frac{4m\pi^2k^2e^4}{2h^2}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

The cluster of constants is called the Rydberg Energy, R$_E$ and has a value of -13.6 eV. When an electron makes a transition from one energy level to another it is assumed that the energy is given off as a photon of light and hence for the transition from m = 3 to n = 2 the energy of the photon is:

$$\Delta E_{mn} = E_m - E_n = 13.6\left(\frac{1}{n^2} - \frac{1}{m^2}\right) = 13.6\left(\frac{1}{4} - \frac{1}{9}\right) = 1.889eV = hc/\lambda$$

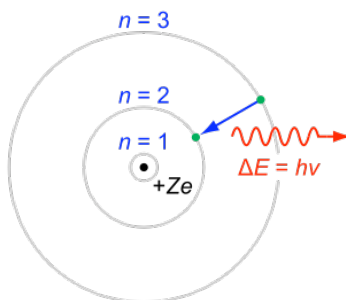Figure 1

Figure 1 shows the transition from n = 3 to n = 2 and has a corresponding energy of 1.889 eV and the corresponding photon emitted has a wavelength of 656 nm. This is called the H$_\alpha$ line of the Balmer Spectrum. The experimental and theoretical results are remarkably good and because of the close agreement with experiment the Bohr model was the first successful theoretical framework that described the spectral lines of hydrogen. The Rydberg constant was empirically determined in 1890 by Rydberg using spectroscopic data. Bohr theory allows for a theoretical determination of the constant and is one of the most compelling aspects of the theory. The visible lines in the Balmer spectrum are shown in figure 2.
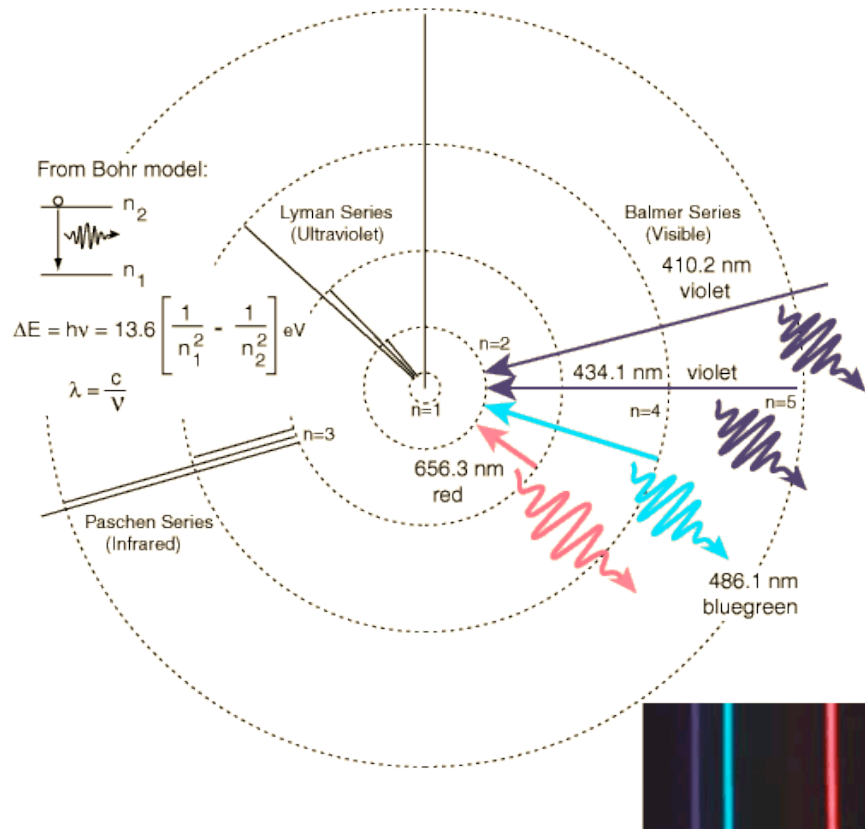
Figure 2 – Balmer Spectrum

From Bohr model:

$$\Delta E = h\nu = 13.6 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] eV$$

$$\lambda = \frac{c}{\nu}$$

Lyman Series (Ultraviolet)

Balmer Series (Visible)

410.2 nm violet

434.1 nm violet

656.3 nm red

Paschen Series (Infrared)

486.1 nm bluegreen

n=1   n=2   n=3   n=4   n=5

Figure 3

Figure 3 shows the Lyman Series, the Paschen Series, and the Balmer series of spectral lines. As can be seen the Lyman Series represents transitions from an excited state to the ground state (n=1) and produces photons in the uv range; the Paschen series represents transitions from excited state to n = 3 state and the photons are in the infrared range. The Balmer Series are transitions to the n = 2 state and lie in the visible range.

The Bohr model is a stunning example of a simple model that explains experimental observations with remarkable accuracy and introduced concepts that are foundational to quantum mechanics.

The Spectroscope

Spectroscopy is one of the most powerful experimental techniques available. There are many types of spectroscopy; optical, energy, impedance, and many more. Basically spectroscopy is breaking down a signal into its constituent parts by energy, frequency, or some other appropriate quantity. Optical spectroscopy takes a light signal and finds its components in terms of wavelength or frequency, which both amount to the same thing. The origins of the art date to Newton who showed that sunlight could be broken into a continuous spectrum of colors. Fraunhofer observed absorption lines in the spectrum (though by Newton to be continuous) of the sun as shown in figure 4.
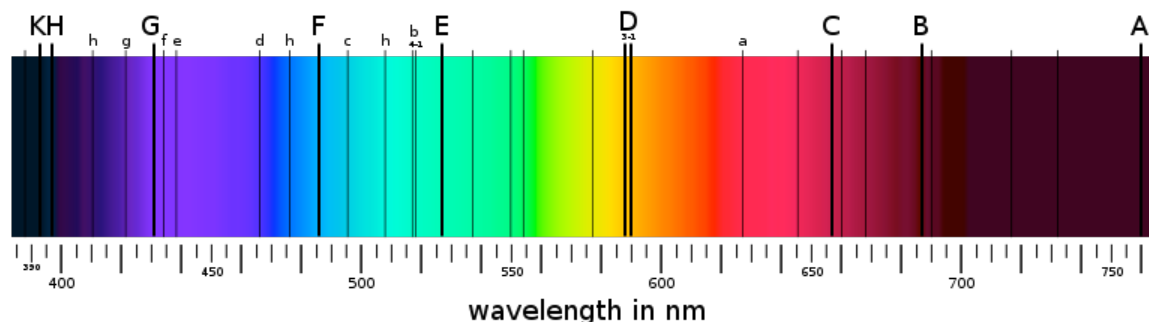

Figure 4

These absorption lines occur because of elements on/near the surface of the sun absorb these frequencies of light. The spectral lines given off by Hydrogen as it transactions from an excited state to a lower energy state are called emission spectra. In this experiment an electrical current is conducted through a tube containing hydrogen gas and in the process the hydrogen atoms transition to high-energy states and then decay back to a lower energy state with the emission of a photon of light. An optical spectrometer will be used to determine the wavelength of the four visible spectral lines in the Balmer series referred to H_α, H_β, H_γ, and H_δ. The H_α is the transition from n = 3 to n = 2, H_β is the transition from n = 4 to n = 2, and so on.

The optical spectrometer that will be utilized in the experiment employs a diffraction grating to refract the light through an angle that depends on wavelength. The operation of a diffraction grating can be understood using the Huygens-Fresnel principle, which states that when light passes through an aperture the aperture may be considered a point source of light. In passing through a diffraction grating each grating aperture acts as a light source. The multiple light sources (one for each aperture) will interfere with one another. Constructive interference will occur when the waves differ by a integral multiple of wavelengths. Diffraction gratings are of two main types; reflective gratings and transmittance gratings. The former actually reflect the light while the latter consist of a number of aligned slits with a separation distance d. To obtain the fundamental diffraction relationship consider two adjacent slits in a diffraction grating separated by a distance d. In figure 5 a wave front is approaching the diffraction grating and the light that is scattered can be considered a point source of light. The figure on the right hand side shows the light leaving two adjacent slits in the grating and as can be seen from the figure the path length difference is dsinθ. If constructive interference is to occur then the path lengths must differ by an integral number of wavelengths or:

$$m\lambda = d \sin \theta$$

where m is an integer and referred to as the order of the line. In our case we will be dealing with only first order diffraction patterns so m = 1. Generally speaking the diffraction gratings are specified in terms of the number of slits per cm or inch.
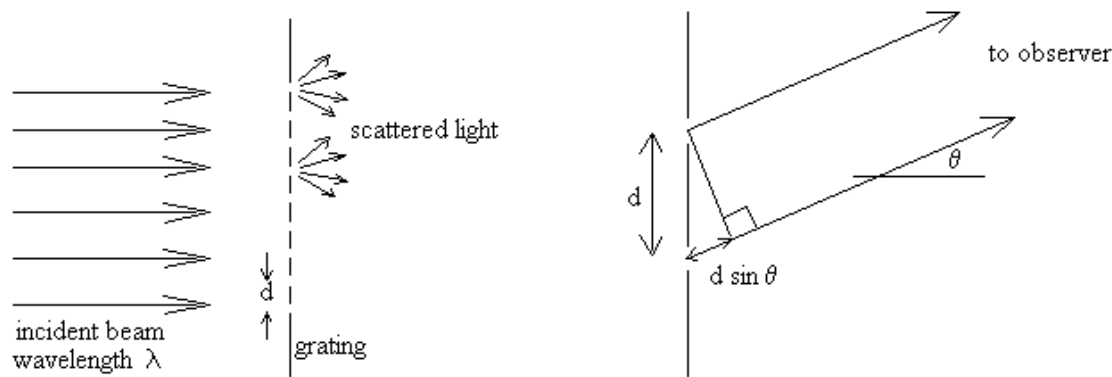
Figure 5

For example a grating might be 20,000 lines per inch, in which case the spacing is d = 1/N, where N is the number of lines per inch. Some useful links for various calculations may be found at:
http://hyperphysics.phy-astr.gsu.edu/Hbase/phyopt/gratcal.html
http://www.ee.byu.edu/photonics/diffraction.phtml

For a grating with 20,000 lines per inch the value of d = 1.27 μm = 1.27 x 10^{-6} m. The calculation is simple but the links are a good reference and are useful for checking yourself.

In the case of the Balmer series the longer the wavelength the greater the angle of diffraction; for the H$_\alpha$ line, whose wavelength is 656 nm the angle of diffraction can be found by using the expression above with m = 1.

$$\sin\theta = \frac{\lambda}{d} = \frac{656 \times 10^{-9}}{1270 \times 10^{-9}} = .517 \Rightarrow \theta = 31.1^o$$

Experimental Procedure

The grating constant must be determined and you will do in two ways. Knowing the number of lines per inch or mm calculate d directly as shown above. The second method involves looking at a spectral line with a known wavelength and finding the corresponding value of d. You will use either the sodium Doublet or some other reference line. Compare the two results. There should be good agreement. If not repeat the procedure until you achieve close agreement between the two methods. Which method is more accurate? Justify your argument.

The hydrogen source is powered by a 75 watt high voltage power supply. The leads to the hydrogen tube are insulated; however take care not to touch the tube after the power has been turned on. Inappropriate handling could result in a serious electrical shock. Align the aperture of the spectroscope so that a bright line can be seen when the telescope and source are in line. The diffraction grating should be perpendicular to the light source. If handling of the diffraction grating is necessary be careful not to touch the grating itself; handle the grating by touching only the outer edges of the grating holder.

In order to see the spectral lines the lights will have to be extinguished. Slowly rotate the telescope until you see a spectral line. Start with the H$_\alpha$, the red line that will be at the greatest angle. It takes a bit of practice to locate the spectral lines and one difficulty is not looking straight down the telescope. A method to be sure you are looking down the scope is to use a small flashlight to generate ambient light that will allow you tell whether or not you are looking directly down the axis of the telescope. Once you have located a line use the cross hairs in the telescope to align the scope and the spectral line. Record the angle for the H$_\alpha$ line. Next

locate the H$_\alpha$ line on the other side and repeat the procedure. For each line you will have an angle to the right and one to the left. It is a good idea to take three measurements (three right and three left) for each spectral line. The value of λ may then be found as:

$$\lambda = d\left(\frac{|\sin\theta|_R + |\sin\theta|_L}{2}\right)$$

The reason for taking a right and left measurement is that errors tend to cancel one another from left to right. You may wish to automate your calculations using a spread sheet, which is an excellent idea, however be sure that you convert degrees to radians or you will generate a great deal of heart ache for yourself. The conversion from degrees to radians is: (θ/360) x 2π = .017θ.

If you do not know how to read the vernier on the spectroscope follow this link: http://www.upscale.utoronto.ca/PVB/Harrison/Vernier/Vernier.html

Using your calculated data calculate the Rydberg constant for each wavelength and the find the average value of the Rydberg constant. Find your percent error. Is your error consistent with possible experimental error? The Rydberg constant is 1.097371568525 x 10^{+7}m^{-1}.

After completing the above you will be given a unknown substance. Identify as many spectral lines as you can and see if you can identify the unknown element or substance.
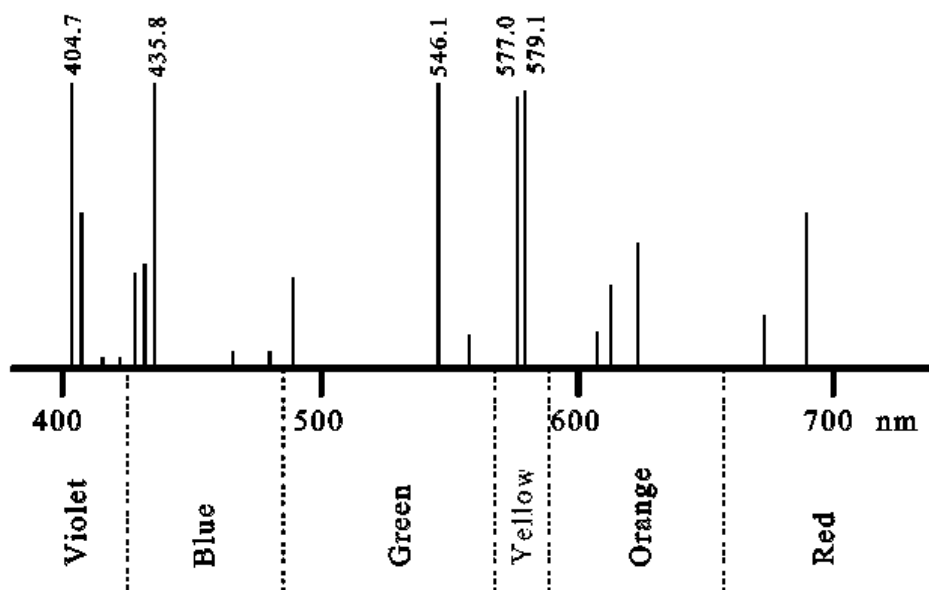

Figure 6

The spectrum above is for mercury, which is one of the unknowns, however for your unknown go to the link below. Select an element from the applet and you will find the wave lengths by pointing at the spectral lines.

http://astro.u-strasbg.fr/~koppen/discharge/discharge.html

Another useful link (NIST National Institute of Standards and Technology):

http://physics.nist.gov/PhysRefData/ASD/lines_form.htm

Exercises

1. An experimenter is measuring the H_α line of hydrogen and finds that the error associated with the measurement is ±5 minutes of arc. What will the corresponding uncertainty in the wavelength be? Evaluate the variation about the angle you found experimentally.

2. If all of the wavelengths found in your experiment were 1% less than the expected value what might you conclude? What kind of error is likely involved in this sort of situation?

3. Find a general expression for the Balmer series. Calculate the Balmer wavelengths based on your value of the Rydberg constant. Calculate using the accepted value of the Rydberg constant.

4. Using the accepted value of the Rydberg $1.097371568525 \times 10^{+7} m^{-1}$ find the wavelengths of the Lyman series and the Paschen series. How well do these agree with the experimental values? Note you may find using a spreadsheet to calculate these useful. (Figure 7 may be useful.)
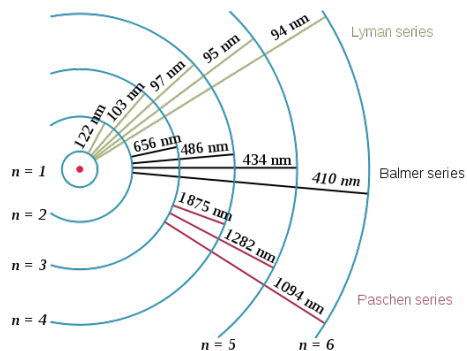

Figure 7

The photoelectric was explained by Einstein in 1905 with his bold assertion that E = hν, where h is Plank's constant and ν is the frequency of the photon. The explanation offered by Einstein resolved all of the issues related to the photoelectric effect and won him the 1921 Nobel Prize in physics. In 1924 De Broglie proposed that particles could have wave like properties.

If you think about Einstein's assertion as it relates to the photoelectric effect it soon becomes clear that the electromagnetic wave, which is distributed in space, interacts with an electron ejecting it from a metal surface in the case of the photoelectric effect. In other words the electron absorbs all of the electron's energy just as if the photon were a particle colliding with the electron in an inelastic collision. The photon is behaving as if it were a particle. The Compton effect is another illustration of a photon participating in an event usually associated with particle interaction. The Compton effect occurs when incident x-ray or γ-rays interact with the electrons in matter. The incident γ interacts with an electron and both particles are scattered. Compton derived the formula for this type of interaction by assuming conservation of energy and momentum using the relativistic expression for energy and assuming the energy of the γ is given as hν.

It is worth noting that the wave description and particle description are in some senses a dichotomy. To make this clear consider a vibrating string whose kinetic energy is distributed along the length of the string and is, for a particular element of string dx, proportional to the square of the element's velocity. A particle on the other hand has all of its kinetic energy localized at the site of the moving mass. Similar arguments can be made regarding the potential energy of each. Now when a photon interacts with an electron, as in the case of the photoelectric, effect the energy of the photon, which was apparently distributed, suddenly appears at the site of the interaction and manifests as if it were a particle. The Compton effect is similar and again the photon behaves as if were particle like when it interacts with the electron.

Great success what achieved by Einstein and later Compton by ascribing these particle like properties to entities that were treated as wave like in classical theory. Along comes de Broglie and in essence asserts what is good for the gander is good for the goose. If a photon can manifest as a particle a particle can manifest as a wave. Or to put it differently photons and electrons have both wave like and particle like characteristics and this is often stated as the "wave particle duality". Many folks get exercised about this and make it much more of a mystery than it is – remember physics is about modeling observations and we are trying to model very complex interactions with very simple models so it is no wonder that now and again things don't fit into our simplistic modeling efforts.

Einstein made the assertion that the energy of a photon was proportional to the photon's frequency, namely E = hν. De Broglie assumed that a particle, such as an electron, could be though of in wave like terms and asserted that the wave length of a particle was associated with the particle's momentum, p. The de Broglie hypothesis is: $\lambda p = h$. This equation is reminiscent of equation λν = c for waves. Perhaps de Broglie was let to hypothesis by realizing the wavelength of a pool ball had to be very, very short and that momentum was intimately related to velocity. (Note according to the De Broglie hypothesis the wave length of a pool ball with a momentum of 1 MKS unit is 6.626 x 10^{-34} m, which is impossible to measure and thus a pool ball is localized in space, however an electron moving at c/2 has a wave length of approximately 4.7 x 10^{-12} m or 4.7 x 10^{-3} nanometers. This is a wavelength that differs by only a few orders of magnitude from visible light.

If the de Broglie hypothesis is correct this means that electrons should display wave like properties and this experiment illustrates the wave like nature of electrons through diffraction of electrons. The method to illustrate that something is a wave is to experimentally find its wavelength, which may be accomplished by diffracting the wave using a diffraction grating. A diffraction grating was used to see the Balmer lines of the hydrogen spectrum. In the text above the relationship between the grating lines and the angle of diffraction was given as:
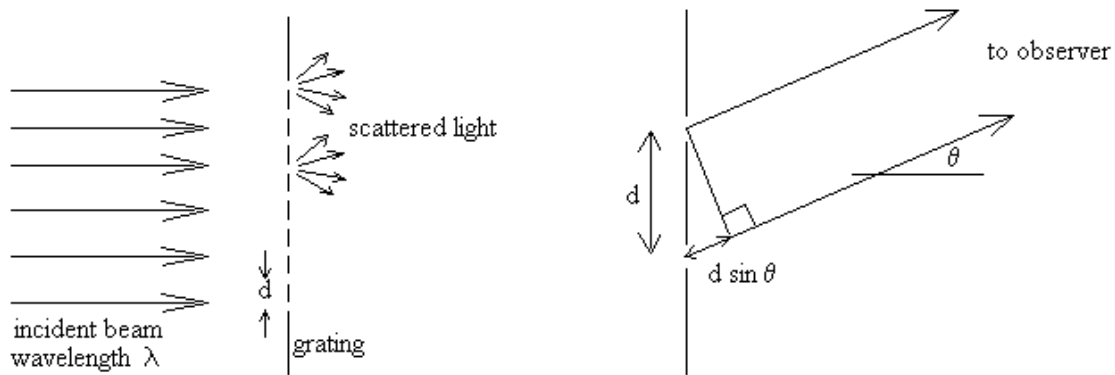
Figure 1

Recall that the Huygens method was employed as explained above to derive the equation that characterized diffraction from multiple slits.

$$m\lambda = d \sin\theta$$

The wave length of an electron, according to our calculation above, is on the order of picometer, that is a wave length of the order of 10^{-12} m. Assuming a first order diffraction m = 1 in the equation then:

$$\sin\theta = \frac{\lambda}{d}$$

This equation shows that λ and d must be of the same order other wise $\sin\theta$ is either undefined ($\lambda > d$) or the angle is so small it cannot be used to measure the wavelength (d $>> \lambda$) because the diffraction line is too close to the central maximum. Thus a diffraction grating with spacing on the order 10^{-12} m or in the picometer range is required. Note – you might argue that higher order diffraction lines might get you out of the woods; however this leads to other difficulties from an experimental point of view. The problem is of course that as m goes up amplitude goes down and thus such an attempt is a road to nowhere. Finding a suitable diffraction grating is not a problem; in 1912 it occurred Max von Laue that a regular crystal with well defined structure might be a good three-dimensional diffraction grating for the investigation of x-rays. William Bragg, a British physicist, used the idea put forth by von Laue and developed Braggs law that essentially says a regular crystalline solid may be thought of having a series of reflection planes each of which will reflect x-rays. The figure below illustrates the case:
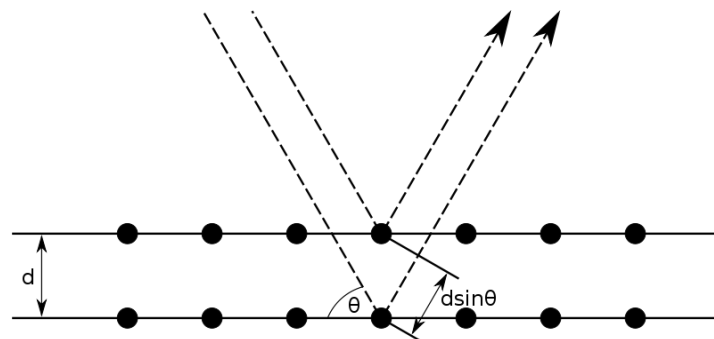

Figure 2

The difference in path length is now 2dsinθ so that like the case for a two-dimensional this difference is set equal to m λ, or Bragg's law is:

$$m\lambda = 2d\sin\theta$$

The analysis of x-ray diffraction is used to investigate the geometry of crystal structure and the spacing between crystal planes is on the order of Angstroms or 10^{-10} meters, therefore such structures are suitable as diffraction gratings for low energy electrons.

Wavelength of an Electron

Using the de Broglie relationship the wavelength of an electron is:

$$\lambda = \frac{h}{p}$$

If an electron is accelerated through a potential difference of V volts it acquires a kinetic energy given as eV = ½ mv². So:

$$v = \sqrt{\frac{2eV}{m}} \quad and \quad mv = p = \sqrt{2meV}$$

$$and \quad \lambda = \frac{h}{\sqrt{2meV}} = \sqrt{\frac{150}{V}}$$

Where the last expression has been obtained by evaluating the constants h, m, and e and where λ is in Angstroms and V is in volts.

The Experiment – Thompson Method

A number of experiments were performed to verify the de Broglie hypothesis and in this experiment the method due to John Paget Thompson, son of J. J. Thompson, will be used.

Figure 3

The photo in figure 3 is the original apparatus used by Thompson; the device is called an electron diffraction camera. It houses a small metal target that is bombarded with electrons that are diffracted forming a circular

diffraction pattern on the focal plane of the film. By analysis of the ring pattern Thompson was able to determine the wavelength of the electrons by using the Bragg's law.

In our experiment the target is aluminum foil and the structure of the foil consists of many randomly oriented reflection planes. These planes are generally described with what are referred to as Miller indices; a complete understanding of them is beyond the scope this lab, but their use nonetheless must be employed to analyze the results of the experiment. The important concepts associated with crystal structure, as they relate to the experiment, are twofold; there are a variety of possible reflection planes that may serve as a three dimensional diffraction grating for the electrons and these planes may be specified using the Miller indices. No attempt will be made to derive the equations used in the analysis, however you should understand that there are multiple parallel planes that will refract the electrons and our data will be found using trial values of the Miller indices.