MEASUREMENT OF RAINFALL BY RADAR

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ABSTRACT

Experiments at wavelength 10 cm have verified the theoretical finding that the power obtained from radiation reflected from rain is proportional to Z, the sum of the sixth powers of diameters of raindrops contained in a representative volume. No value has been obtained for the constant of proportionality. Correlation of Z with rainfall intensity R (both at ground level) on logarithmic scales shows that, within a factor of about two, $Z \propto R^2$. Vertical scans of rain storms by radar indicate that variation of rain content with height is moderate and accountable. It may be possible therefore to determine with useful accuracy the intensity of rainfall at a point quite distant (say 100 km) by the radar echo from that point.

1. Introduction

Studies and possible studies of radar echoes from rain cover a wide field, as outlined by Bent (1946) and Bemis (1947). They have a direct practical application in revealing the development and motion of storm areas. They are providing an instrument for the analysis in both the horizontal (Maynard, 1945; Wexler, 1947a; 1947b) and the vertical (Byers and Coons, 1947) of storms and precipitation mechanisms. All these interrelated studies call for a clear understanding of the basic physical process of scattering (Wexler and Swingle, 1947). That process is an interesting one in its own right. The large number of presumably independent scatterers involved (10^{10} or more) affords a statistical reliability not available in the more usual radar processes.

This paper is primarily concerned with the basic physical process, but a fortuitous relation between the precipitation quantity important to radar ($Z = \Sigma D^6/V$) and one that is more relevant to meteorology (rate of rainfall) may prove useful in practical and research applications.

2. Application of scattering theory

(a) *Radiation scattered back from one drop.*—Application of scattering theory to this problem is largely due to J. W. Ryde.[2] He first defines a scattering function for a single drop, thus:

$$I_{r1} = \frac{I_1}{r^2}\frac{B_1}{r^2} = \frac{I_1 B_1}{r^4}, \quad (1)$$

where I_{r1} is the intensity at the radar of radiation scattered back by one drop at distance r, I_1 is the intensity of the outgoing radiation at unit distance from the transmitter, and B_1 is a scattering function for the radiation (wave length) and drop (size and dielectric constants). This relation depends on the assumption that attenuation by the atmosphere and by the rain itself is negligible, as it usually is for 10-cm radiation. For shorter wave lengths, attenuation must be considered, as it is by Wexler and Swingle (1947).

When D/λ, the ratio of the diameter of the drop to the wave length, is small enough (<0.03 for water, <0.015 for ice), Ryde finds

$$B_1 = \frac{\pi^4}{4\lambda^4}\frac{(\eta^2-1)^2 + \eta^2\chi^2[2(\eta^2+1)+\eta^2\chi^2]}{(\eta^2+2)^2+\eta^2\chi^2[2(\eta^2-2)+\eta^2\chi^2]}D^6, \quad (2)$$

where η and χ are the indices of refraction and absorption, respectively, of water.

When reported laboratory measurements of η and χ are substituted, it appears that further simplification

[1] All formerly of the Canadian Army Operational Research Group.

[2] J. W. Ryde, "Echo intensities and attenuation due to clouds, rain, hail, sand and dust storms at centimetre wavelengths," Research Laboratories of the General Electric Company (British), 13 October 1941.

by neglecting absorption is permissible. Then $\chi = 0$, so (2) becomes

$$B_1 = \frac{\pi^4}{4\lambda^4}\frac{(\eta^2-1)^2}{(\eta^2+2)^2}D^6, \quad (3)$$

as in Rayleigh's theory of scattering.

If Saxton's[3] laboratory values of η are substituted, $(\eta^2-1)^2/(\eta^2+2)^2$ is found to be nearly constant at about 0.95 for water between 0C and 40C. (According to laboratory values for ice by Lamb and Dunsmuir,[4] it is nearly constant at 0.17 for ice between −40C and 0C.) Although this applies not only at $\lambda = 10$ cm but also down to $\lambda = 1.6$ cm, the functions in (2) and (3) are not so appropriate when λ is less than 10 cm, because of the condition that D/λ must be small.

(b) *The scattering region.*—Suppose that the radar emits energy at a uniform rate from time $t = 0$ to $t = h/c$ where c is the speed of light. This defines h as the length of the wave train. Denote by r the distance from which scattered radiation can reach the radar at time t. Then

$$2r_{\max} = tc,$$

$$2r_{\min} = (t - h/c)c = tc - h.$$

Therefore,

$$r_{\max} - r_{\min} = \tfrac{1}{2}h.$$

Apart from the directional effect of the antenna, the region from which scattered radiation can reach the radar at time t consists of all that space lying at distances between $r_{\min}$ and $r_{\max}$, a region between two concentric spheres and having volume

$$\tfrac{4}{3}\pi(r^3_{\max} - r^3_{\min}).$$

If $h \ll r$, this volume is approximately

$$4\pi r^2\cdot\tfrac{1}{2}h = 2\pi r^2 h. \quad (4)$$

(c) *Uniform distribution of radiation and rain.*—One drop at range r gives an intensity at the radar of

$$I_{r1} = \frac{I_1 B_1}{r^4} = \frac{I_1\pi^4}{4r^4\lambda^4}\frac{(\eta^2-1)^2}{(\eta^2+2)^2}D^6 \quad (5)$$

(equations 1 and 3).

Let Z be the sum of sixth powers of diameters of the drops in unit volume, and $I_{r\Sigma}$ the intensity at the radar due to all the drops in unit volume. The unit must be taken large enough for the number of drops to be large.[5] Assume a completely random distribution of drops in space. Then, since the intensities from many independent sources are additive,

$$I_{r\Sigma} = \frac{I_1\pi^4}{4r^4\lambda^4}\frac{(\eta^2-1)^2}{(\eta^2+2)^2}Z. \quad (6)$$

The total volume from which scattered radiation can reach the radar at time t is very nearly $2\pi r^2 h$ (equation 4). Suppose that this volume were uniformly irradiated (I_1 the same in all directions) and uniformly filled with drops (Z constant throughout this region between concentric spheres). Let I_r denote the total intensity at the radar due to scattering from drops. Then

$$I_r = 2\pi r^2 h I_{r\Sigma} = I_1\frac{\pi^5 h}{2r^2\lambda^4}\frac{(\eta^2-1)^2}{(\eta^2+2)^2}Z. \quad (7)$$

(d) *Concentrated beam.*—If the intensity (power per unit area) of outgoing radiation is uniform in all directions, then

$$I_1 = P_0/(4\pi),$$

where P_0 is the output power of the radar. If this equation is substituted in (7),

$$I_r = P_0\frac{\pi^4 h}{8r^2\lambda^4}\frac{(\eta^2-1)^2}{(\eta^2+2)^2}Z.$$

Now suppose the radiation is concentrated, i.e., distributed nonuniformly with direction. The same power will pass through the same scattering medium; the same fraction will be scattered back. The only changes will be these:

1. The uniform distribution of rain need no longer extend in all directions, so long as it exists at distances $(r - \frac{1}{2}h)$ to r in the direction of the beam.

2. A mirror-type or directional receiving antenna becomes feasible.

With a mirror-type receiving antenna,

$$P_r = I_r A,$$

where P_r is the received power and A is the effective area of the antenna. Intensity can now be eliminated completely from (7):

$$P_r = P_0\frac{\pi^4 A h}{8r^2\lambda^4}\frac{(\eta^2-1)^2}{(\eta^2+2)^2}Z. \quad (8)$$

Two of the assumptions made in deriving the above relation, that attenuation is negligible and that D/λ is small enough, hold fairly well for 10-cm radiation but not for 3-cm or 1-cm. The third assumption, that the cross section of the beam is uniformly filled with rain at the given range, is considered at length at the end of this article; it is probably valid out to the range at which the top of the radar beam reaches the freezing level.

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[3] J. A. Saxton, "The anomalous dispersion of water at very high radio frequencies in the temperature range 0° to 40°C," 6 April 1945.

[4] J. Lamb and J. Dunsmuir, "The dielectric properties of ice at wavelengths of 3 and 9 cm," Ministry of Supply Extra Mural Research, 5 March 1945.

[5] The quantity $Z = (\Sigma D^6)/V$, where V is volume of air, is somewhat similar to the volume or mass of rain per unit volume of air, $M = \frac{1}{6}\pi(\Sigma D^3)/V$.

3. The experiment

(a) *General plan.*—During the summer of 1946, a Royal Canadian Air Force microwave-height-finder (MHF) radar was used. The antenna produced a horizontally polarized beam 8° in horizontal and 1.5° in vertical extent (to half power).

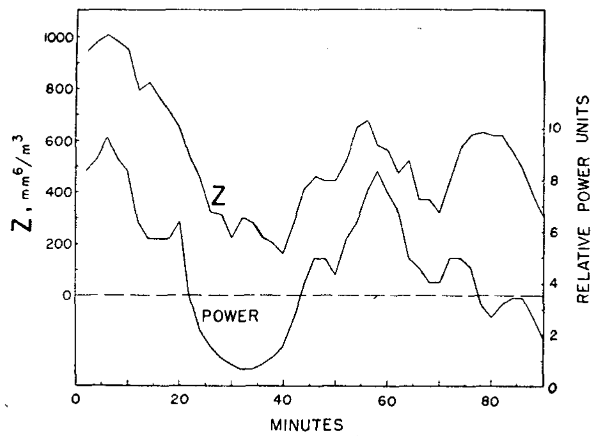

FIG. 1. Received power measured at the radar and Z measured on the ground under the radar beam at range 8.8 km, both plotted against time with ordinates offset for convenience in comparing the variations in the two quantities.

The beam of the radar was placed on a fixed azimuth and at a mean elevation of 1.5° to clear the local permanent echoes. Measurements of Z and R were made at the ground, at range 8.8 km, where the beam extended from 120 m to 350 m above the ground. Allowance was made for the time (never more than one minute) required for the drops to fall from the beam to the ground.

At the radar, signal strength was measured by keeping the A-scope signal at constant height by means of a gain control in the receiver calibrated with an S-band signal generator. Some absolute measurements were attempted. Sample results, for 16 May 1946, are shown in fig. 1.

(b) *Measurement of Z.*—The quantity Z has been determined (for rain near the ground) for a few hundred cases by catching samples of rain on filter papers which had been treated with a trace of powdered gentian-violet dye. The drops left stains, the diameters of which were a function of the diameters of the original raindrops. This function was determined experimentally.

The papers used were Whatman No. 1, diameter 24 cm. They were exposed in folders of corrugated cardboard, approximately fifty such folders being kept in use. Usually two papers were exposed simultaneously every two minutes, so as to obtain at least 100 drops in the sample. (During a series of measurements, two people exposed the papers in pairs every two minutes. A third person removed the exposed papers and reloaded the folders.) The time of exposure varied from 30 seconds in very light rain to 3 seconds in heavy rain. If rainfall was measured simultaneously with a sensitive rain gauge, the time of exposure of the papers did not need to be measured, as will be explained below.

The diameters of the stains on the dyed paper were measured with a transparent ruler which was calibrated to read directly the diameters of the original raindrops. In establishing the relation between stain diameter and drop diameter, large drops were obtained from a micropipette. For small drops, since uniform ones were difficult to obtain, a different method was employed. It was noted that small drops of water formed practically perfect hemispheres on a waxed surface. The diameters of these hemispherical drops were measured by means of a traveling microscope; then the drops were taken up carefully by the dyed filter paper. The calibration data (fig. 2) fall along the two-thirds-power curve anticipated by simple theory.

The fact that the speed of fall of a raindrop varies with its size must be taken into account in going from a sample of rain arriving at a horizontal surface to a sample existing in space. Terminal speeds for all sizes of raindrops are available in meteorological handbooks. Measurements by E. D. Smith[6] have been used here.

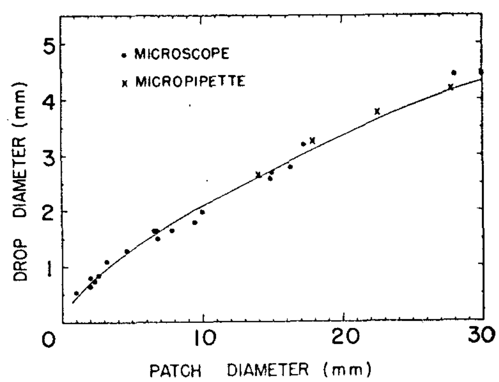

FIG. 2. Calibration curve for drop-size measurement using Whatman No. 1 filter paper. The curve through the points is the locus of: (drop diameter) = (patch diameter)$^{2/3}$.

Both the scattering quantity Z and the rate of rainfall R (depth per unit time) can be calculated from the filter-paper data. It was found more accurate, however, to measure R directly. The value of R obtained directly and the filter-paper data then served to determine Z.

Sensitive rain gauges were improvised from 12-inch funnels and graduated cylinders, so that R could be evaluated over a one- or two-minute interval. These

[6] E. D. Smith, "Notes on rain drop size and attenuation of one way microwave radio transmission," Third Conference on Propagation, 16–18 November 1944, Washington, D. C.

gauges were usually used in pairs about 6 feet apart, the cylinders being read every half minute. Latterly a Short and Mason rain gauge was used, with enlarged funnel and high-speed recording.

The quantity Z was computed from the samples on filter paper according to the following development: Let

$N_D\,\delta D$ be the number of drops of diameter between $D - \frac{1}{2}\delta D$ and $D + \frac{1}{2}\delta D$ in unit volume,
$N_{GD}\,\delta D$ be the number falling through unit horizontal area in unit time,
$N_P\,\delta D$ be the number falling on a horizontal paper of area A in time t, and
V_D be the vertical speed of a drop of this size.

Then

$$Z = \Sigma N_D D^6 \delta D,$$

$$N_{GD} = N_D V_D \quad \text{and} \quad N_P = AtN_{GD}.$$

Hence

$$Z = \frac{\Sigma(N_P D^6/V_D)\delta D}{At}. \tag{9}$$

But t is difficult to measure, and some statistical improvement is obtained by determining At from R and the relation

$$R = \tfrac{1}{6}\pi\,\Sigma N_{GD} D^3\,\delta D = (\tfrac{1}{6}\pi\,\Sigma N_P D^3\,\delta D)/(At).$$

Hence (9) becomes

$$Z = \frac{R\,\Sigma(N_P D^6/V_D)\delta D}{\frac{1}{6}\pi\,\Sigma N_P D^3\,\delta D}. \tag{10}$$

The accuracy of the individual determinations of Z has been assessed by comparing for many occasions the two values obtained from a pair of simultaneously exposed filter papers. In all of the 185 available cases, the ratio of the greater to the smaller had a median value of 1.3. From this it has been deduced that if many papers were exposed simultaneously the probable deviation of any paper from the mean would be 21 per cent. Drop-size distributions obtained from these samples are in good agreement with those of Laws and Parsons (1943) for similar rates of rainfall.

4. Results

(a) *The proportionality of P_r and Z.*—On 12 July 1946 the rainfall varied in less than one hour from 1 to 33 mm/hr (Z from 350 to 64,000 mm^6/m^3) yet was free from sudden discontinuities. Only relative values of P_r were obtained, but the completion of the observations in less than one hour kept down variations in radar performance. The data from this experiment are given in fig. 3. The points in this figure fall reasonably well about a locus $P_r \propto Z$. (Precise fitting indicates that $P_r \propto Z^{1.1}$ is closer.) Surprisingly, however, P_r is just as nearly proportional to R^2 as to Z. This is an extremely useful finding, although it does not appear to be theoretically significant.

(b) *The constant of proportionality.*—With range and output power constant, the received power has been found proportional to Z in agreement with (8). The constant of proportionality involves four parameters:

λ was known from frequency measurements to be 10.7 cm,
A was taken as 60 per cent of the actual aperture, i.e., $A = 4.5$ m^2,
h from earlier adjustment was taken as 0.3 km,
P_0, average power during the pulse, from specifications, 200 kw.

Measurements of P_r indicated that the received power was much less than (8) would indicate, but unmeasured transmission losses through the equipment, both outward bound and returning, would reduce P_r by a considerable factor. Measurements on different days were reasonably consistent, and there did not appear to be any rapidly varying losses.

(c) *The empirical relation between Z and R.*—At the same time that the received power was found to be nearly proportional to Z, it was also found to be just as nearly proportional to R^2, where R is the rainfall (fig. 3). That the latter relationship is not peculiar to a particular day becomes evident when $\log Z$ is plotted against $\log R$ for all the samples of rain taken

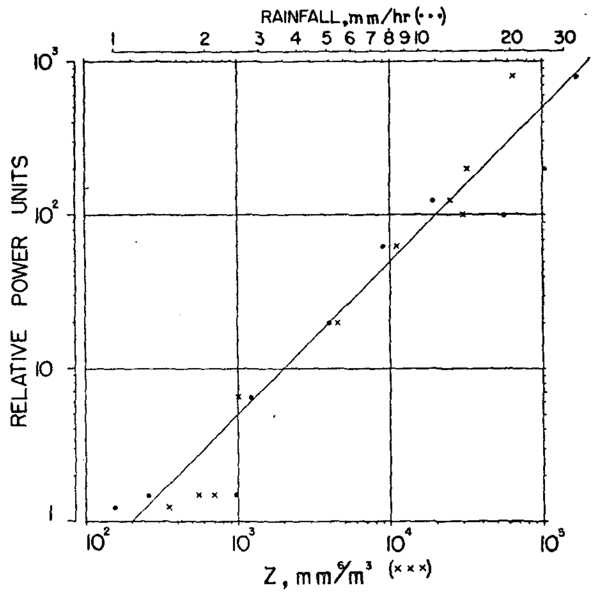

FIG. 3. Received power measured at the radar plotted against rainfall (upper scale) and Z (lower scale), both measured on the ground under the radar beam at range 8.8 km. Scales are all logarithmic. The scale intervals for log R are just twice those for log Z. The scale for log R has been shifted so that the same locus serves both sets of observational points.

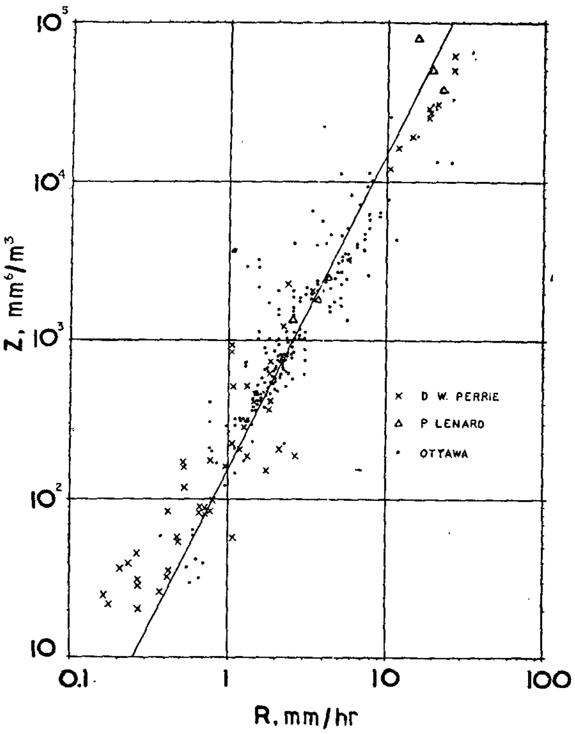

FIG. 4. Log Z plotted against log R for many simultaneous measurements made on several different days by different observers, including D. W. Perrie (Canadian Meteorological Service, research at Clinton, Ontario, 1945) and Lenard (1904). The line which has been drawn is not the best locus; it has been chosen for its simple square-law relation.

in the course of this work (fig. 4). The considerable data, obtained on several different occasions with different types of weather, and in several different locations, fall in a reasonably narrow band about the locus

$$Z = 190\, R^{1.72}, \quad \text{or} \quad R = 0.048\, Z^{0.58}, \qquad (11)$$

where Z is in mm^6/m^3 and R in mm/hr. Part of the spread in fig. 4 is attributable to inaccuracies in the determination of Z; the probable error in this determination was calculated above as 21 per cent.

Some correlation between Z and R has been noted previously by Wexler (1947a). While the data which he analyzed did not show such close correlation or such a simple relationship, Wexler suggested that in the future it might be possible to use this relationship to measure the rate of rainfall by radar.

(d) *Proposed definition of 'radar rainfall.'*—Apparently the rate of rainfall[7] R can be determined with useful accuracy by measuring Z by radar. In putting the empirical relation to practical use, it would be convenient to introduce a new quantity, R_R, 'radar rainfall,' defined by a relation of the form of (11), but with simpler numbers, particularly in the index. The following definition is suggested:

$$R_R = 0.08\, Z^{\frac{1}{2}}, \qquad (12)$$

where R_R is in mm/hr and Z in mm^6/m^3. (If R_R is to be in inch/hr, the defining relation becomes

$$R_R^2 = 10^{-5}\, Z,$$

with Z still in mm^6/m^3.)

The line actually drawn in fig. 4 is the locus of (12), not (11). The ratio of the actual rainfall to radar rainfall is generally near unity, rarely less than $\frac{1}{2}$ or greater than $1\frac{1}{2}$, as may be seen from fig. 4.

If (12) is combined with (8), a relation is obtained between the radar rainfall R_R and other radar quantities. Introducing practical units and the numerical value of η, this relation is

$$R_R = 74{,}000\, \lambda^2 (AhP_0)^{-\frac{1}{2}} r P_r^{\frac{1}{2}}, \qquad (13)$$

where λ is the wave-length in cm and A is the effective area of the antenna in m^2, r and h are the range and the length of the wave train in km, and P_0 and P_r are transmitted and received power both in the same units. The radar rainfall is seen to vary slowly with the radar parameters. It is directly proportional to the range and to the amplitude of the received signal (i.e., the square root of the received power). The proportionality to amplitude has experimental verification, but the proportionality to range has not, and assumes negligible attenuation of the radiation in the atmosphere; the experimental constant of proportionality remains uncertain.

If the parameters given above for the equipment used in this experiment are substituted in (13), and if 10^{-12} watts is assumed as the smallest signal that can be detected, then the minimum rainfall detectable at a given range is

$$R_{\min} = 1.63 \times 10^{-2}\, r, \qquad (14)$$

where R is in mm/hr and r in km. If the relation is applicable at the ranges involved, a rainfall of 1.63 mm/hr should be detectable at 100 km, and approximately a rainfall of 0.1 inch/hr at 100 mi. In present practice, rainfalls greater by a factor seven, roughly, are required.

(e) *Vertical distribution.*—The amplitude of the radar echo from rain 8.8 km from the radar and 250 m above the earth has been found to be proportional to the rate of rainfall at the ground a fraction of a minute later, when the rain that was giving the echo had had time to fall to the ground.

It is thought that the same proportionality will be found, somewhat less precisely, at greater ranges. For instance, if the vertical extent of the beam is from $\frac{1}{2}°$

[7] From the same rain samplings, it has been found that M, the mass *of rain* in unit volume of air, and R, the rate of rainfall, are well correlated with the locus $M = 80\, R^{0.88}$, where M is in mg/m^3 and R in mm/hr. Combining this with (11), one finds $Z = 0.020\, M^{2.08}$. Thus received radar power should be approximately proportional to M^2. But it must be kept in mind that M is the mass *of rain* in unit volume. The relation would be quite incorrect in cloud if M were taken there to denote all free water including cloud drops, which would increase M substantially and Z hardly at all.

to $1\frac{1}{2}°$ and if the freezing level is above 3,000 m, then 100 km is estimated as the range to which one might hope for useful proportionality.

This estimate is based on observations made with the MHF radar of the vertical distribution of echoes from rain. These observations revealed two types of echo structure, one associated with continuous rain and the other with showers, the two types being found either separately or mixed in almost any proportion.

In the continuous-rain structure (fig. 5) the echo varies only gradually with range. The variation with height is slight from the ground up nearly to the freezing level. The echo is much stronger for a band 200–400 m thick at or near the freezing level. Above this band, the echo is faint and uncertain. Tibbles and Eon[8] correlated radar observations with precipitation samplings made simultaneously from an aircraft under observation by the radar. They found rain below the bright band (i.e., layer of stronger echo), snow above, and melting snow in the band. Characteristics of this band are discussed at greater length by Byers and Coons (1947).

In the shower structure (fig. 6), the echo varies suddenly with range. Vertical uniformity is better; the echo pattern is made up of nearly vertical streaks, whose departure from the vertical can be explained by wind shear. The tops of the showers, as indicated by radar, are very variable in height, but for rain showers they are mostly above 3,000 m. Since strong vertical air currents must play a considerable part in these showers, the accumulation of great quantities of rain aloft might be suspected. Continuous observations of individual showers over periods of the order of an hour indicate that such accumulations are short-lived, however. The matter is being given further study; meanwhile it is hoped that the mean rate of descent of rain in a shower is not greatly different from the rate in still air.

There is always a time lag between the time of the radar observations and the arrival of the observed rain at the earth's surface. Considering a beam extending from $\frac{1}{2}°$ to $1\frac{1}{2}°$ above the horizon, at range 100 km the radar beam would be at an average height of 2,000 m above the earth's surface. In still air, as for continuous rain, rain would reach the earth from this height in about 6 minutes. In showers, the time of descent would vary somewhat, say by a factor two either way. If the shower were moving 60 km/hr, the rain would reach the earth somewhere between 3 km and 9 km from the map position at which it was measured by radar. Thus, for showers, the proportionality of rate of rainfall and radar signal at range 100 km might only be valid when averaged over areas of 50 km² to 100 km². Apart from this variation, both

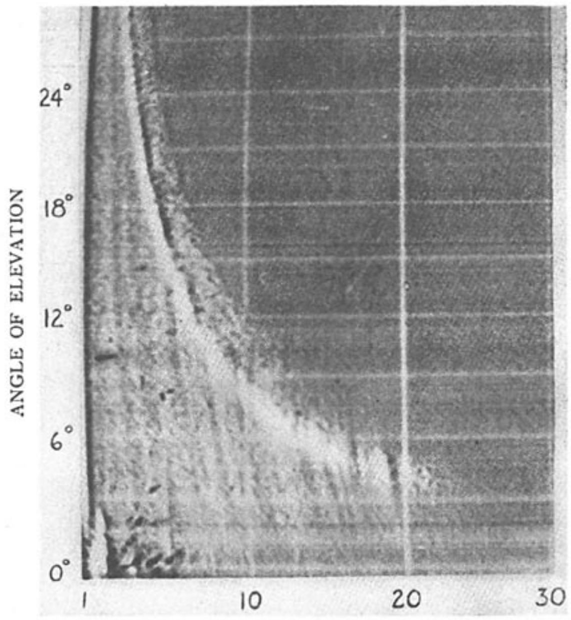

Fig. 5. Continuous rain with bright band, 8 June 1947, MHF radar at Ottawa, Canada.

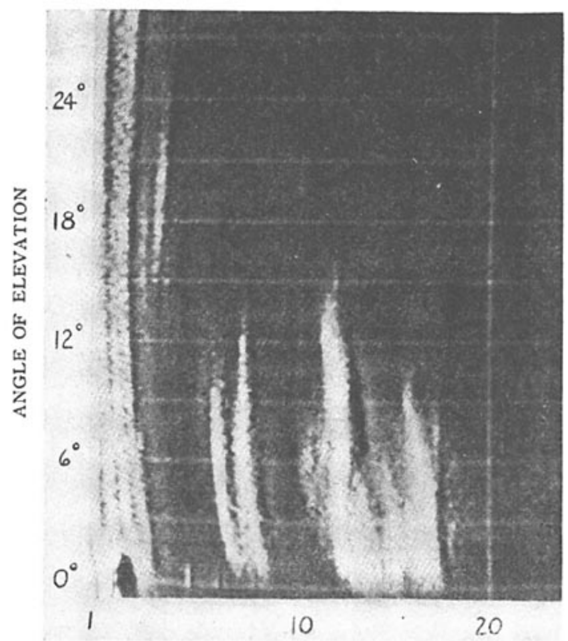

Fig. 6. Rain showers associated with a cold front, 11 June 1947, MHF radar at Ottawa, Canada.

showers and continuous rain appear to be sufficiently uniform with height, up to the freezing level, that proportionality between rate of rainfall at the ground and the amplitude of the radar signal may be hoped for out to the range at which the top of the radar beam reaches the freezing level.

Acknowledgment.—The work reported was part of a larger project undertaken by the Canadian Army

[8] L. G. Tibbles and L. G. Eon, C.A.O.R.G. research at Ottawa, 1945, not published.

Operational Research Group, with extensive co-operation from the National Research Council of Canada, the Royal Canadian Air Force, and the Canadian Meteorological Service.

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