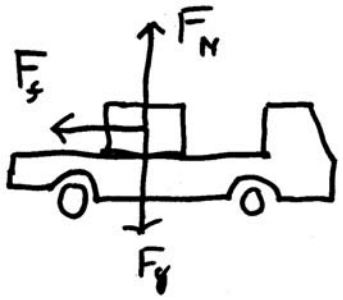
Fall 2006 Exam 3 Solutions
Please Note there are two 4a's and 4b's because of the different exam versions.

1a)



$$F_f = \mu_s F_N = ma$$

$$\mu_s mg = ma$$

$$a = \mu_s g = .4(9.8\ m/s^2) = 3.92\ m/s^2$$

1b)

$$W = \vec{F} \cdot \Delta\vec{r} = (45\hat{i} + 30\hat{j}) \cdot (20\hat{i} + 10\hat{j})$$

$$\boxed{W = (900 - 300)J = 600J}$$

$$W = F \Delta r \cos\theta$$

$$\cos\theta = \frac{W}{F\Delta r} \qquad F = (45^2 + 30^2)^{1/2} = (\vec{F}\cdot\vec{F})^{1/2} = 54.1\ N$$

$$\Delta r = (20^2 + 10^2)^{1/2} = 22.4\ m.$$

$$\boxed{\cos\theta = \frac{600J}{54.1\ N\ 22.4\ m} = .495}$$

$$\theta = \cos^{-1}(.495) = 60.3°$$

1c)

$$K_i = \frac{1}{2} m v_i^2 = \frac{1}{2} m \vec{v}_i \cdot \vec{v}_i$$

$$K_i = \frac{1}{2} m \left(4.00\hat{i} + 3.00\hat{j}\right) \cdot \left(4.00\hat{i} + 3.00\hat{j}\right)$$

$$K_i = \frac{1}{2} (2.4 \times 10^{-3} kg)(16 + 9)$$

$$K_i = .03 \text{ Joules}$$

$$W_{Net} = \Delta K = W_{AR} = K_f - K_i$$

$$K_f = \frac{1}{2} (2.4 \times 10^{-3} kg)(10.89 + 4) = .018 \text{ Joules.}$$

$$W_{AR} = .018 J - .03 J = -.012 J.$$

1d)

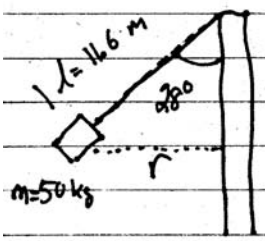
$$F_{gs} = G \frac{m_1 m_2}{r^2} = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2} \frac{(5.94 \times 10^{24} kg)\, 5.35 \times 10^{-26} kg}{6.8 (6.371 \times 10^6 m)^2}$$

$$F_{gs} = 5.2 \times 10^{-25} N$$

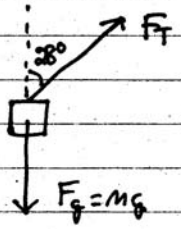
$$F_{g100} = F_{gs} \frac{R_E^2}{(R_E + h)^2} = 5.22 \times 10^{-25} N \frac{(6.371 \times 10^6)^2}{(6.371 \times 10^6)^2}$$

$$F_{g100} = \cancel{4.5 \times 10^{-25}} = 5.06 \times 10^{-25} N$$

Problem 2.



a)



b) $F_{Ty} = F_T \cos 28$

$\Sigma F_y = F_{Ty} - F_g = 0$

$F_T \cos 28 = mg$

$F_T = \frac{mg}{\cos 28} = \frac{50\,kg\,(9.8\,m/s^2)}{\cos(28)} = 555\,N$

c) $r = l \sin\theta = 11.6 \sin 28$

$r = 5.446\,m$

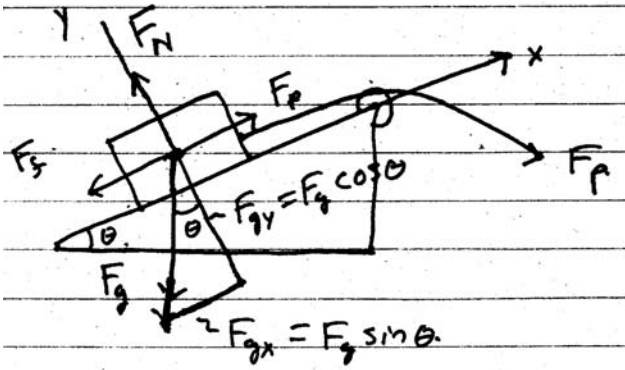
$\Sigma F_c = F_{Tx} = [illegible] 28 = [illegible]$

$v = \left(\frac{F_T \sin 28\, r}{m}\right)^{1/2} = \left(\frac{555 \sin 28\ 5.446\,m}{50\,kg}\right)^{1/2}$

$v = 5.3\ m/s$

d) $F_T \sin 28 = mv^2/r \qquad F_T = \frac{mg}{\cos 2}$

$mg \tan\theta = mv^2/r \quad \therefore \quad v = \left(rg \tan\theta\right)^{1/2}$ No



$$\Sigma F_y = F_N - F_{gy} = 0 \quad \therefore F_N = mg\cos\theta$$

$$\Sigma F_x = F_p - F_s - F_{gx} = 0$$

$$F_p = F_s + F_{gx} = \mu_s mg\cos\theta + mg\sin\theta$$

$$F_p = .4\,(500\ kg)(9.8\ m/s^2)\cos(10) + 500kg\ 9.8\ m/s^2 \sin$$

$$F_p = 2781\ N \quad \ldots$$

$$\Sigma F_x = F_p - F_s - F_{gx} = ma.$$

$$a_x = \frac{F_p - \mu mg\cos\theta - mg\sin\theta}{m} = a$$

$$\frac{2781\ N - .2(500)\,9.8\cos 10 - 500(9.8)\sin 10}{500} = a$$

$$a_x = 1.93$$

4) a) $\cancel{K_i}^{0} + U_{gi} + \cancel{U_{si}}^{0} = \cancel{K_f}^{0} + \cancel{U_{gf}}^{0} + U_{sf}$

$U_{sf} = U_{gi}$

$\frac{1}{2} k x_f^2 = mg y_i$

$x_f = \left(\frac{2m}{k} g y_i \right)^{1/2}$.

$x_f = \left(\frac{2 \cdot 10\,kg}{2250\,N/m} 9.8\,m/s^2 \cdot 3\,m \right)^{1/2} = .5112.\ m.$

b $\cancel{K_i}^{0} + U_{gi} + \cancel{U_{si}}^{0} + W_f = \cancel{K_f}^{0} + \cancel{U_{gf}}^{0} + U_{sf}$

$W_f = U_{sf} - U_{gi}$

$F_f \cdot \Delta r = -\mu mg \cdot \Delta r = U_{sf} - U_{gi}$

$= \frac{1}{2} k x^2 - mg y_i$

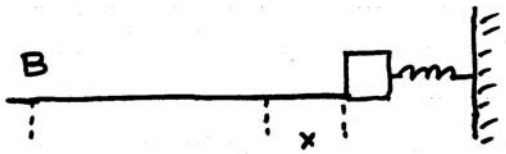
$= \frac{1}{2} (2250\,N/m)(.3\,m)^2 - 10kg\ 9.8\,m/s^2\ 3m$

$W_f = -192.75$

$\mu = -\frac{W_f}{mg \Delta r} = -\frac{-192.75}{10kg\ 9.8\,m/s^2\ 6m}$

$\mu = .328$

4a)



$m = .5\ kg$

$k = 450\ N/m$

$V_B = 12\ m/s$ $\qquad V_i' = 0$

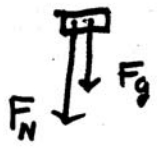
$K_B + U_{sB} = K_i' + U_{si}' \qquad K_i' = 0 \quad U_{sB} = 0$

$\frac{1}{2} k X_i'^2 = \frac{1}{2} m V_B^2$

$X_i'^2 = \left[\frac{m}{k} V_B^2\right]^{1/2}$

$X_i' = \left[\frac{.5\ kg}{450\ N/m} (12\ m/s)^2\right]^{1/2} = .4\ m$

4b



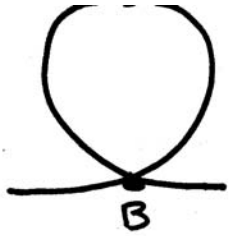
At top $\Sigma F_c = F_N + F_g = mV^2/r$

minimum V corresponds to $F_N = 0$

$F_g = mg = mV^2/r$

$V = \sqrt{gr} = \sqrt{9.8\ m^2/s^2} = 3.13\ m/s$

4c.



$V_B = 12\ m/s$

$W_F = \Delta K + \Delta U_g$

~~$W_F + K_B$~~

$W_F + K_B + \cancel{U_{gB}}^{0} = K_f + U_{gf}$

$W_F = K_f - K_B + U_{gf}$

~~$W_F = \frac{1}{2}(.5\,kg)(12\,m/s)^2 - \frac{1}{2}(.5\,kg)(4\,m/s$~~

$W_F = \frac{1}{2}(.5\,kg)(4\,m/s)^2 - \frac{1}{2}(.5\,kg)(12\,m/s)^2 + .5\,kg\,(9.8\,m/s^2)2$

$W_F = -22.2$ Joule.

4d) $W_F = \vec{F} \cdot \vec{\Delta r} \Rightarrow \quad \Delta r = \frac{W_F}{F} = \frac{-22.2}{\pi r} = -7.1\ m.$