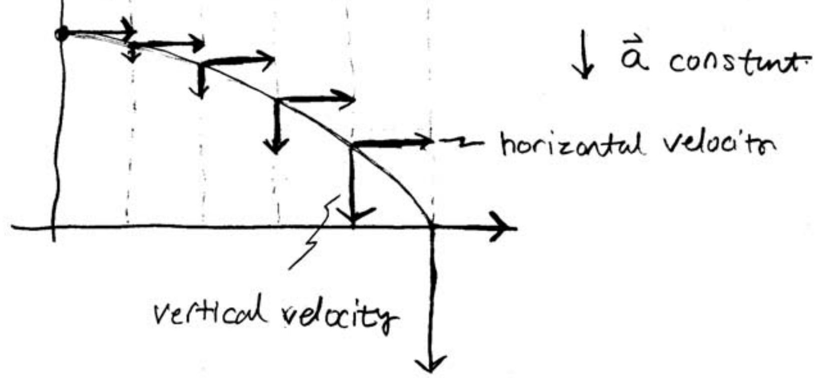
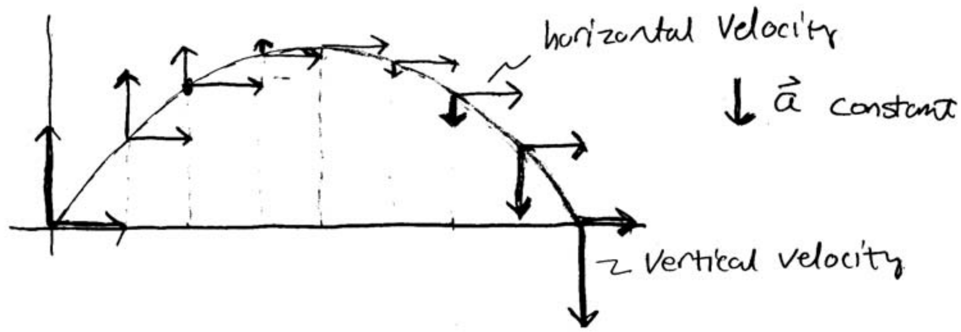
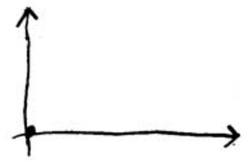
3-Question 2

⇒ Velocity and acceleration vectors for initially horizontal projectile





3.5)



$\vec{r}_i = 0\hat{i} + 0\hat{j}$

$\vec{v}_i = 5\,m/s\,\hat{i} + 0\,\hat{j}$

$\vec{a} = 0\,\hat{i} + 3\,m/s^2\,\hat{j}$

a) $\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}\,t^2$

$\vec{r}_f = 5\,m/s\,\hat{i}\,t + 1.5\,m/s^2\,\hat{j}\,t^2$

$\vec{V}_f = \vec{V}_i + \vec{a}_i t$

$\vec{V}_f = 5\,m/s\,\hat{i} + 3\,m/s^2\,\hat{j}\,t$

b) $\vec{r}_f(2) = 5\,m/s\,(2s)\,\hat{i} + 1.5\,m/s^2\,(2s)^2\,\hat{j}$

$\vec{r}_f(2) = 10\,m\,\hat{i} + 6.0\,m\,\hat{j}$

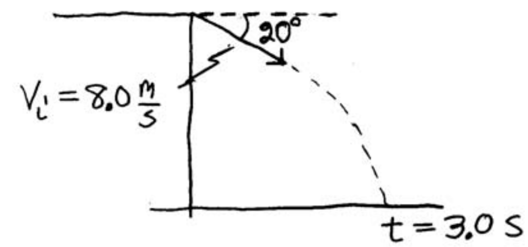
$\vec{V}_f(2) = 5\,m/s\,\hat{i} + 3\,m/s^2\,(2s)\,\hat{j}$

$\vec{V}_f(2) = 5\,m/s\,\hat{i} + 6\,m/s\,\hat{j}$

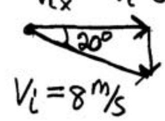
$|\vec{V}_f(2)| = \left[V_{fx}^2 + V_{fy}^2\right]^{1/2} = \left((5\,m/s)^2 + (6\,m/s)^2\right)^{1/2}$

$|\vec{V}_f(2)| = 7.8\,m/s$

3-12)



$V_{ix} = V_i \cos\theta = 8\,m/s \cos(20) = 7.52\,m/s$



$V_{iy} = V_i \sin\theta$

$V_{iy} = 8\,m/s \sin(20) = 2.74 \frac{m}{s}$

$V_{iy} = -2.74\,m/s$

↑ downward

X Problem

$x_i = 0$

$x_f = ?$

$V_{ix} = 7.52\,m/s$

$t = 3.0\,s$

Y Problem

$y_i = ?$

$y_f = 0$

$V_{iy} = -2.74\,m/s$

$V_{fy} = ?$

$a_y = -9.8\,m/s^2$

$t = 3.0\,s$

a) $x_f = x_i + V_{ix}\,t$

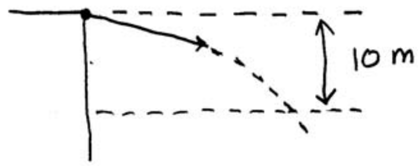
$x_f = 0 + 7.52\,m/s\,(3.0\,s) = 22.6\,m$

b) $\cancel{y_f}^{\,0} = y_i + V_{iy}\,t + \frac{1}{2} a_y t^2$

$y_i = -(-2.74\,m/s)(3.0\,s) - \frac{1}{2}(-9.8\,m/s^2)(3.0\,s)^2$

$y_i = 52.3\,m$

3-12)



$$\Delta y = v_{yi} t + \tfrac{1}{2} a_y t^2$$

We can find V_{fy} and then t or solve the quadratic above.

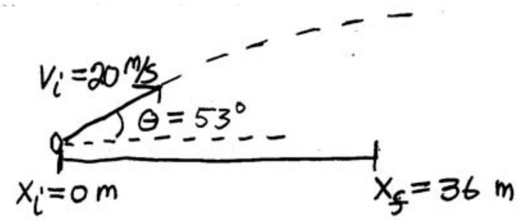
$$-10\,m = -2.74\,m/s\; t - 4.9\,m/s^2\, t^2$$

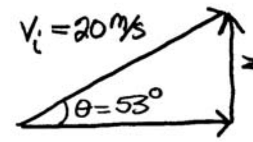
$$4.9\,m/s^2\, t^2 + 2.74\,m/s\; t - 10\,m = 0$$

$$t = \frac{-2.74\,m/s \pm \left((2.74\,m/s)^2 - 4(4.9\,m/s^2)(-10\,m)\right)^{1/2}}{2\,(4.9\,m/s^2)}$$

$$t = \frac{-2.74\,m/s \pm 14.27\,m/s}{9.8\,m/s} = 1.18,\ -1.74\ s$$

3-15)





$V_{ix} = 20\,\text{m/s} \cos(53)$
$V_{ix} = 12\,\text{m/s}$

$V_{iy} = 20\,\text{m/s} \sin(53)$
$V_{iy} = 16\,\text{m/s}$

x problem	y problem
$x_i = 0$	$y_i = 0$
$x_f = 36$ m	$y_f = ?$ $\Rightarrow$ we want it to be 3.05m high or more but we need to find out how high.
$V_{ix} = 12$ m/s	$V_{iy} = 16$ m/s
$V_{fx} = 12$ m/s	$V_{fy} = ?$
$a_x = 0$	$a_y = -9.8\,\text{m/s}^2$
$t = ?$	$t = ?$

Can't solve y problem yet, but get t from x problem.

$$x_f = x_i + V_{ix} t \qquad a_x = 0$$

$$t = \frac{x_f - x_i}{V_{ix}} = \frac{36\,\text{m} - 0\,\text{m}}{12\,\text{m/s}} = 3\text{s}$$

3-15)

$$y_f = y_i + V_{iy}t + \frac{1}{2}a_y t^2$$

$$y_f = 0 + 16\,m/s\,(3\,s) + \frac{1}{2}(-9.8\,m/s^2)(3s)^2$$

$$y_f = 48\,m - 44.1\,m = 3.9\,m$$

$$V_{yf} = V_{yi} + a_y t$$

$$V_{yf} = 16\,m/s + (-9.8\,m/s^2)(3\,s)$$

$$V_{yf} = -13.4\,m/s$$

3-18)

$X_f = 0 + (11.2\ m/s)\cos(18.5°)\ t$

the x position is given by this equation which is consistent with the "constant velocity" equation

$X_f = X_i + V_{xi}\ t$

where $X_i = 0$ & $V_{xi} = 11.2\cos(18.5) = 10.6\ m/s$

$.36\ m = .84\ m + 11.2\ m/s\ \sin(18.5)\ t - \frac{1}{2}(9.8\ m/s^2)t^2$

the y position is given by this equation which is consistent with "constant acceleration" equation

$y_i = .84\ m, \quad V_{yi} = 11.2\ m/s\ \sin(18.5) = 3.55\ m/s$

a) Takeoff $\vec{r}_i = 0\hat{i} + .84\,m\,\hat{j}$

b) Takeoff $\vec{V}_i = 10.6\ m/s\ \hat{i} + 3.55\ m/s\ \hat{j}$

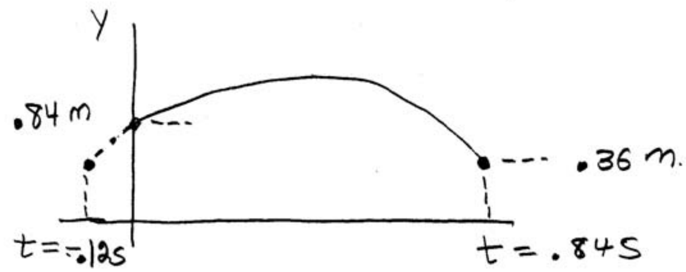
c) solve for time to find how long in the air. Time in The air is determined by The "y" position.

$$0 = .48 + 3.55t - 4.9t^2$$

$$t = \frac{-3.55 \pm \left(3.55^2 - 4(-4.9).48\right)^{1/2}}{2\cdot(-4.9)}$$

3-18

$$t = \frac{-3.55 \pm 4.69}{2(-4.9)} = .84\,s,\ -.12\,s$$



P3.13 Consider the motion from original zero height to maximum height h:

$v_{yf}{}^2 = v_{yi}{}^2 + 2a_y(y_f - y_i)$ gives $0 = v_{yi}{}^2 - 2g(h-0)$ or $v_{yi} = \sqrt{2gh}$

Now consider the motion from the original point to half the maximum height:

$v_{yf}{}^2 = v_{yi}{}^2 + 2a_y(y_f - y_i)$ gives $v_{yh}{}^2 = 2gh + 2(-g)\left(\frac{1}{2}h - 0\right)$ so $v_{yh} = \sqrt{gh}$

At maximum height, the speed is $v_x = \frac{1}{2}\sqrt{v_x{}^2 + v_{yh}{}^2} = \frac{1}{2}\sqrt{v_x{}^2 + gh}$

Solving, $v_x = \sqrt{\frac{gh}{3}}$

Now the projection angle is $\theta_i = \tan^{-1}\frac{v_{yi}}{v_x} = \tan^{-1}\frac{\sqrt{2gh}}{\sqrt{gh/3}} = \tan^{-1}\sqrt{6} = \boxed{67.8^\circ}$.

P3.20 From the instant he leaves the floor until just before he lands, the basketball star is a projectile. His vertical velocity and vertical displacement are related by the equation $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$. Applying this to the upward part of his flight gives $0 = v_{yi}^2 + 2(-9.80\ \text{m/s}^2)(1.85 - 1.02)\ \text{m}$. From this, $v_{yi} = 4.03\ \text{m/s}$. [Note that this is the answer to part (c) of this problem.]

For the downward part of the flight, the equation gives $v_{yf}^2 = 0 + 2(-9.80\ \text{m/s}^2)(0.900 - 1.85)\ \text{m}$. Thus the vertical velocity just before he lands is

$$v_{yf} = -4.32\ \text{m/s}.$$

(a) His hang time may then be found from $v_{yf} = v_{yi} + a_y t$:

$$-4.32\ \text{m/s} = 4.03\ \text{m/s} + (-9.80\ \text{m/s}^2)t$$

or $t = \boxed{0.852\ \text{s}}$.

(b) Looking at the total horizontal displacement during the leap, $x = v_{xi}t$ becomes

$$2.80\ \text{m} = v_{xi}(0.852\ \text{s})$$

which yields $v_{xi} = \boxed{3.29\ \text{m/s}}$.

(c) $v_{yi} = \boxed{4.03\ \text{m/s}}$. See above for proof.

(d) The takeoff angle is: $\theta = \tan^{-1}\left(\frac{v_{yi}}{v_{xi}}\right) = \tan^{-1}\left(\frac{4.03\ \text{m/s}}{3.29\ \text{m/s}}\right) = \boxed{50.8^\circ}$.

(e) Similarly for the deer, the upward part of the flight gives

$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$:

$$0 = v_{yi}^2 + 2(-9.80 \text{ m/s}^2)(2.50 - 1.20) \text{ m}$$

so $v_{yi} = 5.04 \text{ m/s}$.

For the downward part, $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$ yields $v_{yf}^2 = 0 + 2(-9.80 \text{ m/s}^2)(0.700 - 2.50) \text{ m}$ and $v_{yf} = -5.94 \text{ m/s}$.

The hang time is then found as $v_{yf} = v_{yi} + a_y t$: $-5.94 \text{ m/s} = 5.04 \text{ m/s} + (-9.80 \text{ m/s}^2)t$ and

$$\boxed{t = 1.12 \text{ s}}.$$