Q7.11 The total energy of the ball-Earth system is conserved. Since the system initially has gravitational energy *mgh* and no kinetic energy, the ball will again have zero kinetic energy when it returns to its original position. Air resistance will cause the ball to come back to a point slightly below its initial position. On the other hand, if anyone gives a forward push to the ball anywhere along its path, the demonstrator will have to duck.

P7.3 The volume flow rate is the volume of water going over the falls each second:

$$3\text{ m}(0.5\text{ m})(1.2\text{ m/s}) = 1.8\text{ m}^3/\text{s}$$

The mass flow rate is $\frac{m}{t} = \rho\frac{V}{t} = \left(1\,000\text{ kg/m}^3\right)\left(1.8\text{ m}^3/\text{s}\right) = 1\,800\text{ kg/s}$

If the stream has uniform width and depth, the speed of the water below the falls is the same as the speed above the falls. Then no kinetic energy, but only gravitational energy is available for conversion into internal and electric energy.

The input power is $\mathscr{P}_{\text{in}} = \frac{\text{energy}}{\Delta t} = \frac{mgy}{\Delta t} = \frac{m}{\Delta t}gy = (1\,800\text{ kg/s})\left(9.8\text{ m/s}^2\right)(5\text{ m}) = 8.82\times10^4\text{ J/s}$

The output power is $\mathscr{P}_{\text{useful}} = (\text{efficiency})\mathscr{P}_{\text{in}} = 0.25\left(8.82\times10^4\text{ W}\right) = \boxed{2.20\times10^4\text{ W}}$

The efficiency of electric generation at Hoover Dam is about 85% with a head of water (vertical drop) of 174 m. Intensive research is underway to improve the efficiency of low head generators.

P7.7 From leaving ground to the highest point, $K_i + U_i = K_f + U_f$

$$\frac{1}{2}m(6.00\text{ m/s})^2 + 0 = 0 + m\left(9.80\text{ m/s}^2\right)y$$

The mass makes no difference: $\therefore y = \frac{(6.00\text{ m/s})^2}{(2)\left(9.80\text{ m/s}^2\right)} = \boxed{1.84\text{ m}}$

P7.9 Using conservation of energy for the system of the Earth and the two objects

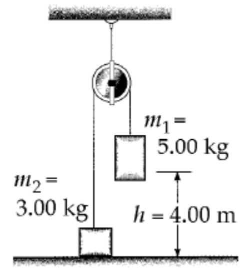

FIG. P7.9

(a) $$(5.00\text{ kg})g(4.00\text{ m}) = (3.00\text{ kg})g(4.00\text{ m}) + \frac{1}{2}(5.00+3.00)v^2$$

$$v = \sqrt{19.6} = \boxed{4.43\text{ m/s}}$$

(b) Now we apply conservation of energy for the system of the 3.00 kg object and the Earth during the time interval between the instant when the string goes slack and the instant at which the 3.00 kg object reaches its highest position in its free fall.

$$\frac{1}{2}(3.00)v^2 = mg\,\Delta y = 3.00g\Delta y$$
$$\Delta y = 1.00\text{ m}$$
$$y_{\max} = 4.00\text{ m} + \Delta y = \boxed{5.00\text{ m}}$$

P7.11 (a) The force needed to hang on is equal to the force F the trapeze bar exerts on the performer. From the free-body diagram for the performer's body, as shown, $F - mg\cos\theta = m\frac{v^2}{\ell}$ or $F = mg\cos\theta + m\frac{v^2}{\ell}$. Apply conservation of mechanical energy of the performer-Earth system as the performer moves between the starting point and any later point:

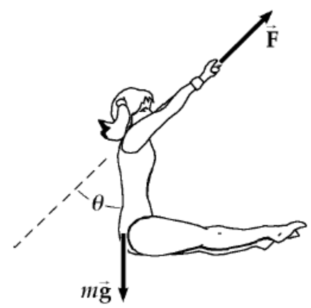

FIG. P7.11

$$mg(\ell - \ell\cos\theta_i) = mg(\ell - \ell\cos\theta) + \frac{1}{2}mv^2$$

Solve for $\frac{mv^2}{\ell}$ and substitute into the force equation to obtain $F = \boxed{mg(3\cos\theta - 2\cos\theta_i)}$.

(b) At the bottom of the swing, $\theta = 0°$ so

$$F = mg(3 - 2\cos\theta_i)$$
$$F = 2mg = mg(3 - 2\cos\theta_i)$$

which gives $\theta_i = \boxed{60.0°}$.

P7.16 Choose the zero point of gravitational potential energy of the object-spring-Earth system as the configuration in which the object comes to rest. Then because the incline is frictionless, we have $E_B = E_A$: $K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$ or $0 + mg(d+x)\sin\theta + 0 = 0 + 0 + \frac{1}{2}kx^2$. Solving for d gives

$$d = \boxed{\frac{kx^2}{2mg\sin\theta} - x}.$$

P7.17 From conservation of energy for the block-spring-Earth system,

$$U_{gt} = U_{si},$$

or

$$(0.250 \text{ kg})(9.80 \text{ m/s}^2)h = \left(\frac{1}{2}\right)(5\,000 \text{ N/m})(0.100 \text{ m})^2$$

This gives a maximum height $h = \boxed{10.2 \text{ m}}$.

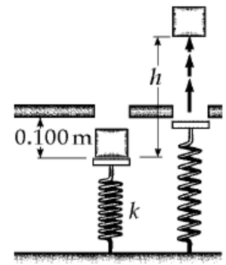

FIG. P7.17

P7.24 We begin with Equation 6.20 for the isolated child-wheelchair-Earth system,

$$\Delta K + \Delta U + \Delta E_{\text{int}} = 0 \qquad (1)$$

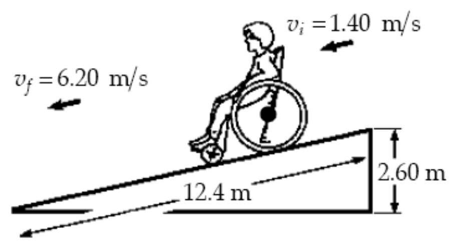

FIG. P7.24

We recognize that the change in potential energy is of two types: gravitational and chemical potential energy stored in the body of the child from past meals. We also recognize that the change in internal energy will be due to the friction force (air resistance and rolling resistance) as the child rolls down the hill. Therefore, we write (1) as,

$$\Delta K + \Delta U_g + \Delta U_{\text{body}} + f_k d = 0 \qquad (2)$$

The change in potential energy in the body of the child will be due to work done within the system by the child on the wheels of the chair. Consequently, the requested answer, the work done by the child, is $W_{\text{child}} = -\Delta U_{\text{body}}$. Therefore, (2) can be expressed as,

$$\begin{aligned}
W_{\text{child}} &= \Delta K + \Delta U_g + f_k d = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + \left(mgh_f - mgh_i\right) + f_k d \\
&= \frac{1}{2}m\left(v_f^2 - v_i^2\right) + mg\left(h_f - h_i\right) + f_k d \\
&= \frac{1}{2}(47.0 \text{ kg})\left[(6.20 \text{ m/s})^2 - (1.40 \text{ m/s})^2\right] + (47.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)(0 - 2.60 \text{ m}) + (41.0 \text{ N})(12.4 \text{ m}) \\
&= \boxed{168 \text{ J}}
\end{aligned}$$

P7.27 (a) $(K+U)_i + \Delta E_{\text{mech}} = (K+U)_f$:

$$0+\frac{1}{2}kx^2 - f\Delta x = \frac{1}{2}mv^2 + 0$$

$$\frac{1}{2}(8.00\ \text{N/m})(5.00\times 10^{-2}\ \text{m})^2 - (3.20\times 10^{-2}\ \text{N})(0.150\ \text{m}) = \frac{1}{2}(5.30\times 10^{-3}\ \text{kg})v^2$$

$$v = \sqrt{\frac{2(5.20\times 10^{-3}\ \text{J})}{5.30\times 10^{-3}\ \text{kg}}} = \boxed{1.40\ \text{m/s}}$$

(b) When the spring force just equals the friction force, the ball will stop speeding up. Here $\left|\vec{\mathbf{F}}_s\right| = kx$; the spring is compressed by

$$\frac{3.20\times 10^{-2}\ \text{N}}{8.00\ \text{N/m}} = 0.400\ \text{cm}$$

and the ball has moved

$$5.00\ \text{cm} - 0.400\ \text{cm} = \boxed{4.60\ \text{cm from the start.}}$$

(c) Between start and maximum speed points,

$$\frac{1}{2}kx_i^2 - f\Delta x = \frac{1}{2}mv^2 + \frac{1}{2}kx_f^2$$

$$\frac{1}{2}8.00(5.00\times 10^{-2})^2 - (3.20\times 10^{-2})(4.60\times 10^{-2}) = \frac{1}{2}(5.30\times 10^{-3})v^2 + \frac{1}{2}8.00(4.00\times 10^{-3})^2$$

$$v = \boxed{1.79\ \text{m/s}}$$

P7.29 (a) The object moved down distance $1.20\text{ m}+x$. Choose $y=0$ at its lower point.

$$K_i + U_{gi} + U_{si} + \Delta E_{\text{mech}} = K_f + U_{gf} + U_{sf}$$

$$0 + mgy_i + 0 + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$(1.50\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m}+x) = \frac{1}{2}(320\text{ N/m})x^2$$

$$0 = (160\text{ N/m})x^2 - (14.7\text{ N})x - 17.6\text{ J}$$

$$x = \frac{14.7\text{ N} \pm \sqrt{(-14.7\text{ N})^2 - 4(160\text{ N/m})(-17.6\text{ N}\cdot\text{m})}}{2(160\text{ N/m})}$$

$$x = \frac{14.7\text{ N} \pm 107\text{ N}}{320\text{ N/m}}$$

The negative root tells how high the object will rebound if it is instantly glued to the spring. We want

$$x = \boxed{0.381\text{ m}}$$

(b) From the same equation,

$$(1.50\text{ kg})(1.63\text{ m/s}^2)(1.20\text{ m}+x) = \frac{1}{2}(320\text{ N/m})x^2$$

$$0 = 160x^2 - 2.44x - 2.93$$

The positive root is $x = \boxed{0.143\text{ m}}$.

(c) The equation expressing the energy version of the nonisolated system model has one more term:

$$mgy_i - f\Delta x = \frac{1}{2}kx^2$$

$$(1.50\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m}+x) - 0.700\text{ N}(1.20\text{ m}+x) = \frac{1}{2}(320\text{ N/m})x^2$$

$$17.6\text{ J} + 14.7\text{ N}x - 0.840\text{ J} - 0.700\text{ N}x = 160\text{ N/m}\,x^2$$

$$160x^2 - 14.0x - 16.8 = 0$$

$$x = \frac{14.0 \pm \sqrt{(14.0)^2 - 4(160)(-16.8)}}{320}$$

$$x = \boxed{0.371\text{ m}}$$

P7.30 The total mechanical energy of the skysurfer-Earth system is

$$E_{\text{mech}} = K + U_g = \frac{1}{2}mv^2 + mgh.$$

Since the skysurfer has constant speed,

$$\frac{dE_{\text{mech}}}{dt} = mv\frac{dv}{dt} + mg\frac{dh}{dt} = 0 + mg(-v) = -mgv.$$

The rate the system is losing mechanical energy is then

$$\left|\frac{dE_{\text{mech}}}{dt}\right| = mgv = (75.0\text{ kg})(9.80\text{ m/s}^2)(60.0\text{ m/s}) = \boxed{44.1\text{ kW}}.$$

P7.32 (a) $$U = -\int_0^x \left(-Ax + Bx^2\right)dx = \boxed{\frac{Ax^2}{2} - \frac{Bx^3}{3}}$$

(b) $$\Delta U = -\int_{2.00\text{ m}}^{3.00\text{ m}} F dx = \frac{A\left[\left(3.00^2\right) - (2.00)^2\right]}{2} - \frac{B\left[(3.00)^3 - (2.00)^3\right]}{3} = \boxed{\frac{5.00}{2}A - \frac{19.0}{3}B}$$

$$\Delta K = \boxed{\left(-\frac{5.00}{2}A + \frac{19.0}{3}B\right)}$$

P7.37 The height attained is not small compared to the radius of the Earth, so $U = mgy$ does not apply; $U = -\frac{GM_1M_2}{r}$ does. From launch to apogee at height h,

$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$: $$\frac{1}{2}M_p v_i^2 - \frac{GM_EM_p}{R_E} + 0 = 0 - \frac{GM_EM_p}{R_E + h}$$

$$\frac{1}{2}\left(10.0\times10^3\text{ m/s}\right)^2 - \left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(\frac{5.98\times10^{24}\text{ kg}}{6.37\times10^6\text{ m}}\right)$$

$$= -\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(\frac{5.98\times10^{24}\text{ kg}}{6.37\times10^6\text{ m} + h}\right)$$

$$\left(5.00\times10^7\text{ m}^2/\text{s}^2\right) - \left(6.26\times10^7\text{ m}^2/\text{s}^2\right) = \frac{-3.99\times10^{14}\text{ m}^3/\text{s}^2}{6.37\times10^6\text{ m} + h}$$

$$6.37\times10^6\text{ m} + h = \frac{3.99\times10^{14}\text{ m}^3/\text{s}^2}{1.26\times10^7\text{ m}^2/\text{s}^2} = 3.16\times10^7\text{ m}$$

$$\boxed{h = 2.52\times10^7\text{ m}}$$

P7.50 $k = 2.50 \times 10^4 \text{ N/m}$, $m = 25.0 \text{ kg}$

$x_A = -0.100 \text{ m}$, $U_g\big|_{x=0} = U_s\big|_{x=0} = 0$

(a) $E_{\text{mech}} = K_A + U_{gA} + U_{sA}$

$$E_{\text{mech}} = 0 + mgx_A + \frac{1}{2}kx_A^2$$

$$E_{\text{mech}} = (25.0 \text{ kg})(9.80 \text{ m/s}^2)(-0.100 \text{ m}) + \frac{1}{2}(2.50 \times 10^4 \text{ N/m})(-0.100 \text{ m})^2$$

$$E_{\text{mech}} = -24.5 \text{ J} + 125 \text{ J} = \boxed{100 \text{ J}}$$

(b) Since only conservative forces are involved, the total energy of the child-pogo-stick-Earth system at point *C* is the same as that at point *A*.

$K_C + U_{gC} + U_{sC} = K_A + U_{gA} + U_{sA}$: $\quad 0 + (25.0 \text{ kg})(9.80 \text{ m/s}^2)x_C + 0 = 0 - 24.5 \text{ J} + 125 \text{ J}$

$$x_C = \boxed{0.410 \text{ m}}$$

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(c) $K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$: $\quad \frac{1}{2}(25.0 \text{ kg})v_B^2 + 0 + 0 = 0 + (-24.5 \text{ J}) + 125 \text{ J}$

$$v_B = \boxed{2.84 \text{ m/s}}$$

(d) *K* and *v* are at a maximum when $a = \sum F/m = 0$ (i.e., when the magnitude of the upward spring force equals the magnitude of the downward gravitational force).

This occurs at $x < 0$ where $k|x| = mg$

or
$$|x| = \frac{(25.0 \text{ kg})(9.8 \text{ m/s}^2)}{2.50 \times 10^4 \text{ N/m}} = 9.80 \times 10^{-3} \text{ m}$$

Thus, $K = K_{\text{max}}$ at $x = \boxed{-9.80 \text{ mm}}$

(e) $K_{\text{max}} = K_A + \left(U_{gA} - U_g\big|_{x=-9.80 \text{ mm}}\right) + \left(U_{sA} - U_s\big|_{x=-9.80 \text{ mm}}\right)$

or
$$\frac{1}{2}(25.0 \text{ kg})v_{\text{max}}^2 = (25.0 \text{ kg})(9.80 \text{ m/s}^2)\left[(-0.100 \text{ m}) - (-0.0098 \text{ m})\right]$$
$$+ \frac{1}{2}(2.50 \times 10^4 \text{ N/m})\left[(-0.100 \text{ m})^2 - (-0.0098 \text{ m})^2\right]$$

yielding $v_{\text{max}} = \boxed{2.85 \text{ m/s}}$

P7.62 (a) At the top of the loop the car and riders are in free fall:

$$\sum F_y = ma_y: \qquad mg \text{ down} = \frac{mv^2}{R} \text{ down}$$

$$v = \sqrt{Rg}$$

Energy of the car-riders-Earth system is conserved between release and top of loop:

$$K_i + U_{gi} = K_f + U_{gf}: \qquad 0 + mgh = \frac{1}{2}mv^2 + mg(2R)$$

$$gh = \frac{1}{2}Rg + g(2R)$$

$$\boxed{h = 2.50R}$$

(b) Let h now represent the height $\geq 2.5R$ of the release point. At the bottom of the loop we have

$$mgh = \frac{1}{2}mv_b^2 \qquad \text{or} \qquad v_b^2 = 2gh$$

$$\sum F_y = ma_y: \qquad n_b - mg = \frac{mv_b^2}{R}(\text{up})$$

$$n_b = mg + \frac{m(2gh)}{R}$$

At the top of the loop, $mgh = \frac{1}{2}mv_t^2 + mg(2R)$

$$v_t^2 = 2gh - 4gR$$

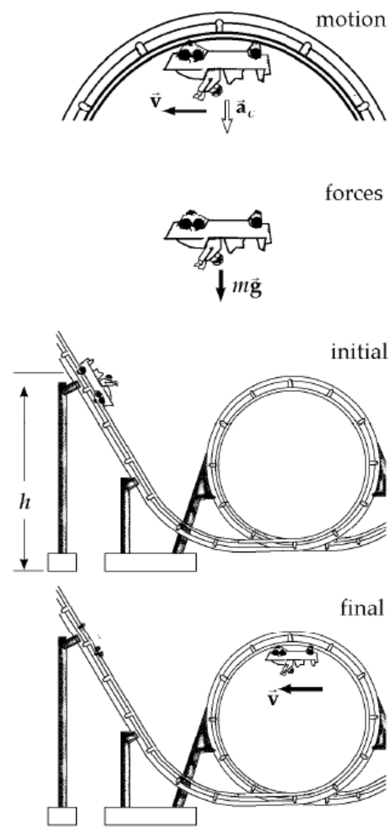

FIG. P7.62

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$$\sum F_y = ma_y: \qquad -n_t - mg = -\frac{mv_t^2}{R}$$

$$n_t = -mg + \frac{m}{R}(2gh - 4gR)$$

$$n_t = \frac{m(2gh)}{R} - 5mg$$

Then the normal force at the bottom is larger by

$$n_b - n_t = mg + \frac{m(2gh)}{R} - \frac{m(2gh)}{R} + 5mg = \boxed{6mg}.$$