

Exam 1: February 13, 2007

Questions and Problems: Provide clear and logical answers to each of the following questions. In question 3 answer 2 of the 3 parts (a-c). Where calculations are required, neatly show all work. You must clearly show all work to receive full credit. Be sure that your answers have the correct units. If you continue your work on another sheet of paper, be sure that it is clearly labeled. Be sure to include FB and other diagrams where appropriate.

1(20 points) The figure to the right shows the height (y) of the rear bumper on my car after the car goes over a small bump. The apparent simple harmonic motion (lack of damping) is indicative of bad shock absorbers. Using the information on the graph:

- Determine the period and frequency of the motion.
- If the mass of the car is 1800 kg, determine the effective spring constant of the car springs.
- Determine the amplitude and phase constant of the motion and write the solution for y as a function of t .
- Determine the total energy of the system.
- Determine the maximum velocity of the bumper.

a) Inspection. Motion repeats in 8 s

$$T = 8 \text{ s}$$

$$f = 1/T = \frac{1}{8 \text{ s}} = .125 \frac{1}{\text{s}} \quad \omega = 2\pi f = .785 \frac{1}{\text{s}}$$

b) $\omega^2 = \frac{k}{m}$ $k = m\omega^2 = m 4\pi^2 f^2 = 1800 \text{ kg} (4\pi^2) (.125 \text{ Hz})^2$

$$k = 1110 \text{ kg/s}^2$$

c) $A = .1 \text{ m}$ inspection from graph

solution $y = A \cos(\omega t + \phi)$ $y(0) = 5 \text{ cm}$

$$\cos(\phi) = \frac{y}{A} = \frac{5}{10}$$

$$\phi = 60^\circ \pi/3 \quad y = 10 \text{ cm} \cos\left(.785 \frac{1}{\text{s}} t - \frac{\pi}{3}\right)$$

$$j) E = U_{s\max} = \frac{1}{2} k x^2 = \frac{1}{2} 1110 \frac{\text{kg}}{\text{s}^2} (.1 \text{ m})^2 = 5.6 \text{ J} \#$$

$$1e) E = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2E}{m}} = \left(\frac{2(5.6 \text{ J})}{1800 \text{ kg}} \right)^{1/2} = .079 \text{ m/s}$$

2 (20 points) A string with mass per unit length $\mu = 2.0 \text{ gm/m}$ is stretched along the positive x-axis with tension 20 N. One end of the string, at $x = 0 \text{ m}$ oscillates up and down at a frequency of 100 Hz with a maximum displacement of 1.0 mm. At $t = 0$, the end of the string is at $y = 0$ ($\phi = 0$).

- What is the wavelength and wave speed of a wave propagating along the string?
- What is the amplitude of the wave?
- What is the string's displacement at $x = 0.50 \text{ m}$ and $t = 15 \text{ ms}$?

$$a) \quad v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{20 \text{ N}}{2 \times 10^{-3} \text{ kg/m}}} = 100 \frac{\text{m}}{\text{s}}$$

$$\lambda = \frac{v}{f} = \frac{100 \text{ m/s}}{100 \text{ 1/s}} = 1 \text{ m}$$

$$b) \quad A = 1 \times 10^{-3} \text{ m}$$

$$c) \quad y = A \sin(kx - \omega t)$$

$$y = 1 \times 10^{-3} \text{ m} \sin(2\pi x - 200\pi t)$$

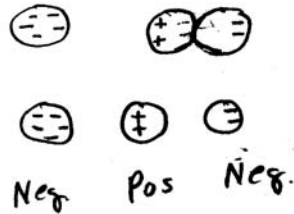
$$\text{Note } k = 2\pi/\lambda = 2\pi \quad \omega = 2\pi f = 2\pi \cdot 100$$

$$y = 1 \times 10^{-3} \text{ m} \sin(2\pi(0.5) - 200\pi(15 \times 10^{-3}))$$

$$y = 1 \times 10^{-3} \text{ m} \sin(-6.28 \text{ rad}) = 1 \times 10^{-3} \sin(-2\pi)$$

$$y = 0$$

3a (10 points) A large negatively charged object is placed on an insulated table. A neutral metallic ball rolls straight toward the object, but stops before it touches it. A second neutral metallic ball rolls along the path followed by the first ball, strikes the first ball, and stops. The first ball rolls forward, but does not touch the negative object. At no time does either ball touch the negative object. What is the final charge on each ball? Explain



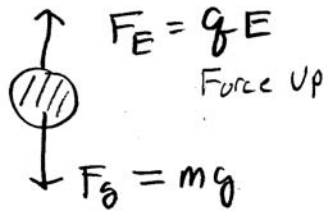
3b (10 points) An aluminum nail has an excess charge of $+3.2 \mu\text{C}$. How many electrons must be added to the nail to make it electrically neutral?

To be electrically neutral must at $q = -3.2 \mu\text{C}$

$$q = -3.2 \mu\text{C} = -N e$$

$$N = -\frac{q}{e} = \frac{3.2 \mu\text{C}}{1.6 \times 10^{-19} \text{C}} = 2 \times 10^{13}$$

3c (10 points) A foam ball of mass 0.150 g carries a charge of -2.00 nC . The ball is placed inside a uniform electric field, and is suspended against the force of gravity. What are the magnitude and direction of the electric field?

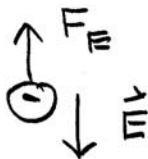


$$qE = mg$$

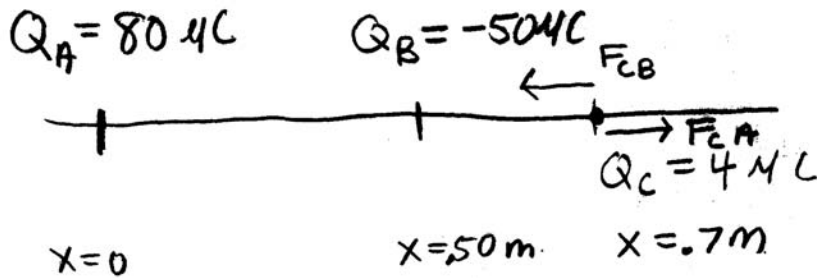
$$E = \frac{m}{q} g$$

$$E = \frac{0.15 \times 10^{-3} \text{ kg} \cdot 9.8 \text{ m/s}^2}{2 \times 10^{-9} \text{ C}}$$

$$E = 7.35 \times 10^5 \text{ N/C}$$



4 (20 points) A charge of $80 \mu\text{C}$ is placed on the x -axis at $x = 0$. A second charge of $-50 \mu\text{C}$ is placed on the x -axis at $x = 50 \text{ cm}$. What is the magnitude of the electrostatic force in N on a third charge of $4.0 \mu\text{C}$ placed on the x -axis at 70 cm ? (For full credit, include a diagram correctly showing the individual forces on the $4.0 \mu\text{C}$ charge.)



$$F_{CA} = k_E \frac{|q_A| |q_C|}{r_{CA}^2} = 8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \frac{|80 \times 10^{-6} \text{C}| |4 \times 10^{-6} \text{C}|}{(0.7 \text{ m})^2}$$

$$F_{CA} = 5.9 \text{ N} \quad \text{to the right}$$

$$F_{CB} = 44.95 \text{ N} \quad \text{to the left} \quad \text{not } r_{CB} = 0.2 \text{ m}$$

$$F = F_{CA} - F_{CB} = 44.95 - 5.9 = 39.05$$

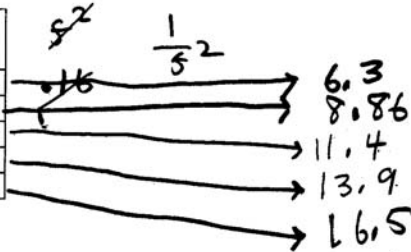
to the
right left

$$\vec{F} = -39.05 \text{ N} \hat{i}$$

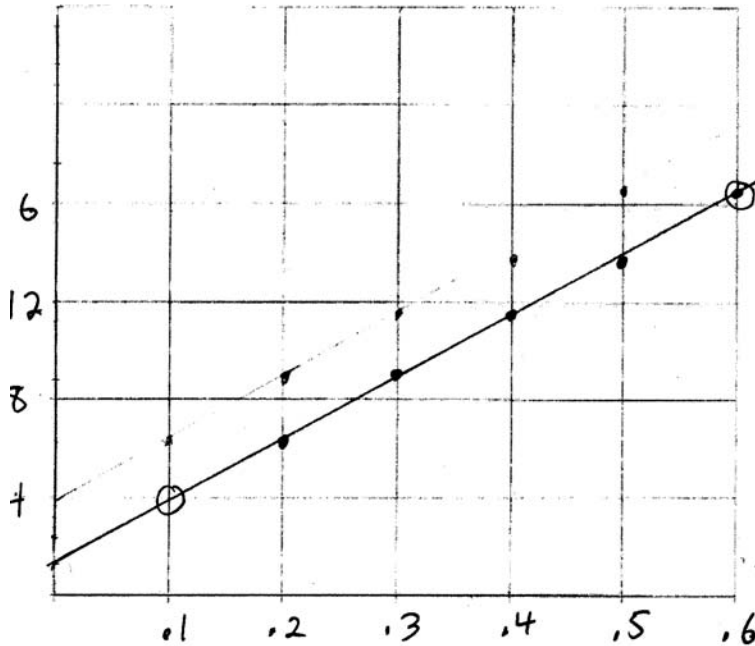
5 (20 points) A mass connected to a spring (mass is not negligible) oscillates horizontally on a frictionless surface. The frequency of the oscillation for a given attached mass is measured and recorded in the table to the right. Theoretically the relationship between the frequency and the mass for a spring whose mass can not be ignored is given by

$$\frac{1}{f^2} = \frac{4\pi^2}{k} m + \frac{4\pi^2}{k} m_{\text{eff}}$$

Mass (kg)	Frequency (Hz)
0.2	0.398724
0.3	0.336247
0.4	0.296384
0.5	0.267953
0.6	0.246805



Using a graph of the data (m as the independent variable on the x axis and some function of the frequency on the y axis), determine the spring constant and effective mass of the spring from the properties of the graph. Clearly show your work and reasoning.



$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{12.5}{0.5} \approx 25$$

$$\text{slope} = \frac{4\pi^2}{k}$$

$$k = \frac{4\pi^2}{\text{slope}} = 1.6 \frac{\text{kg}}{\text{s}^2}$$

$$b = 1.3 = \frac{4\pi^2}{k} m_{\text{eff}}$$

$$m_{\text{eff}} = \frac{1.3 k}{4\pi^2}$$

$$m_{\text{eff}} = 0.052 \text{ kg}$$