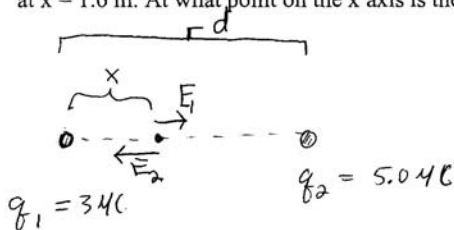


The following are the solutions for Exam II. They are a bit “unrefined”. The methodology is correct, but I corrected a few errors using cross-outs.

Exam 2: March 20, 2007

Questions and Problems: Provide clear and logical answers to each of the following questions. You must answer 3 of the first 4 questions. You must answer 3 of the last 4 questions. Where calculations are required, neatly show all work. You must clearly show all work to receive full credit. Be sure that your answers have the correct units. If you continue your work on another sheet of paper, be sure that it is clearly labeled. Be sure to include FB and other diagrams where appropriate.

1 (20 Points) A charge $q_1 = 3.0 \mu\text{C}$ is located at the origin and a charge of $q_2 = 5.0 \mu\text{C}$ is located at $x = 1.6 \text{ m}$. At what point on the x axis is the electric field equal to zero.



$$x = \frac{-9.6 \pm (9.6^2 - 4(2)(-7.68))^{1/2}}{4}$$

$$x = \frac{-9.6 \pm 12.4}{4} = 0.7 \text{ m}$$

$$E_1 = E_2$$

~~$$k_e \frac{q_1}{r_1^2} = k_e \frac{q_2}{r_2^2}$$~~

$$k_e \frac{q_1}{r_1^2} = k_e \frac{q_2}{r_2^2}$$

$$\frac{q_1}{x^2} = \frac{q_2}{(d-x)^2}$$

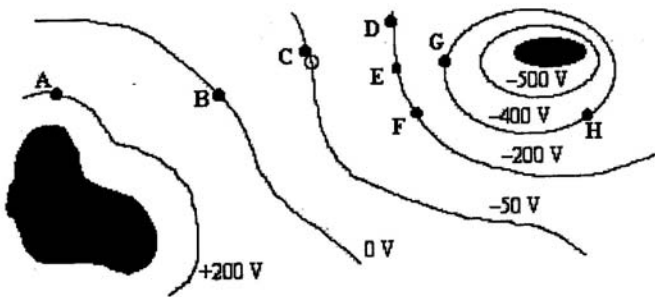
$$q_1 (d^2 - 2dx + x^2) = q_2 x^2$$

$$(q_2 - q_1)x^2 + 2dq_1x - q_1d^2 = 0$$

$$2.4 \text{C} x^2 + 2(1.6 \text{m})3.4 \text{C}x - 3.4 \text{C}(1.6 \text{m})^2 = 0$$

$$2x^2 + 9.6x - 7.68 = 0$$

2 (Points) The sketch shows cross sections of equipotential surfaces between two charged conductors shown in solid black. Points on the equipotential surfaces near the conductors are labeled A, B, C, ..., H.



- What is the magnitude of the potential difference between points A and H?
- What is the direction of the electric field at point E?
- How much work is required to move a $+6 \mu\text{C}$ point charge from B to F to D to A?
Done by the Electric Field
Electric field.

$$a) \quad \Delta V = V_A - V_H = 200\text{V} - (-400\text{V}) = 600\text{V}$$

$$b) \quad E = -\frac{\Delta V}{\Delta x} \text{ points towards } \ominus$$

c) Equal to work from B to A.

$$W_E = -q \Delta V = -q(V_A - V_B) \\ = -6 \mu\text{C} (200\text{V}) = -1200 \mu\text{J} \\ \text{Energy stored.}$$

3) An alpha particle ($q = 2e$, $m = 6.64 \times 10^{-27}$ kg) is shot towards a Uranium nucleus ($q = 92e$) which has a radius of $r = 7.4 \times 10^{-15}$ m. What is the minimum initial kinetic energy needed to just penetrate the nucleus? Given this energy, what would the initial velocity of the alpha particle have to be? *You may think of nucleus charge as being at a single point inside nucleus*

$2e$
 $0 \dots \dots \dots \bigcirc \xrightarrow{92e}$
 $U_i + KE_i = U_f + KE_f$
 $U_i = 0 \neq KE_f = 0$

$$KE_i = U_f = k_e \frac{q_1 q_2}{r}$$

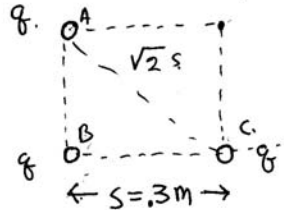
$$KE_i = 8.99 \times 10^9 \text{ N} \frac{(2e)(92e)}{(7.4 \times 10^{-15} \text{ m})}$$

$$KE_i = 5.7 \times 10^{-12} \text{ Joules}$$

$$v = \sqrt{\frac{2KE_i}{m}} = \sqrt{\frac{2 \cdot 5.7 \times 10^{-12} \text{ J}}{6.64 \times 10^{-27} \text{ kg}}}$$

$$v = 4.15 \times 10^7 \text{ m/s} \approx 10\% \text{ speed of light}$$

4) Three charges of $q = 2.0 \mu\text{C}$ are arranged at three corners of a square of side $s = .3$ m (see figure). How much electric potential energy is stored in this charge configuration? How much work is done by the electric force in bringing a $q = -2.0 \mu\text{C}$ charge to the open corner of the square?



$$U_{AB} = k_e \frac{q q}{s} \quad U_{BC} = k_e \frac{q q}{s} \quad U_{AC} = k_e \frac{q q}{\sqrt{2}s}$$

$$U = U_{AB} + U_{BC} + U_{AC} = k_e \frac{q q}{s} \left(2 + \frac{1}{\sqrt{2}} \right) = \frac{8.99 \times 10^9 (2 \times 10^{-6} \text{ C})^2}{.3 \text{ m}} (2.7)$$

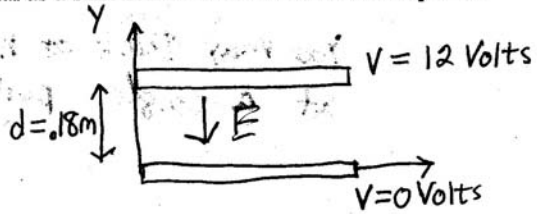
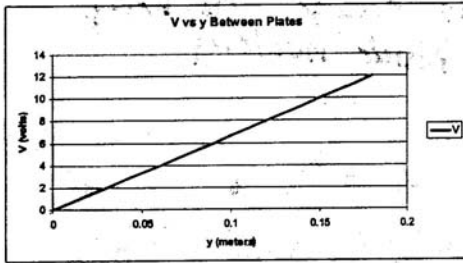
$$= .32 \text{ Joule}$$

$$V = V_A + V_B + V_C = \frac{k_e q}{s} \left(2 + \frac{1}{\sqrt{2}} \right) = 1.62 \times 10^5 \text{ Joule/C}$$

$$U = q_0 V = -2.0 \mu\text{C} (1.62 \times 10^5 \text{ J/C}) = -.324$$

$$W = -U = .324$$

5) Two oppositely charged parallel plates are separated by a distance of $d = .18 \text{ m}$. The top plate is maintained at an electric potential of 12 Volts while the bottom plate has an electric potential of zero Volts. The graph shows the electric potential as a function of distance between the plates.



- a) Determine the electric field at the center of the gap between the plates ($y = 0.09 \text{ m}$).
 b) What is the charge on each of the plates?

$$a) E_y = -\frac{\partial V}{\partial y} =$$

↳ derivative is the slope.

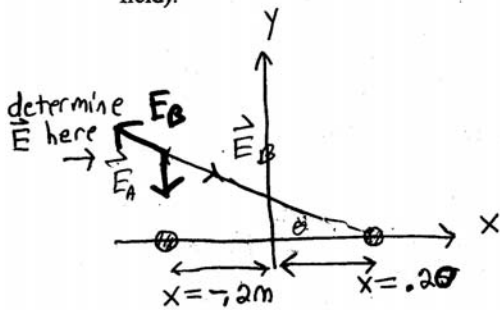
$$E_y = -\frac{12 \text{ V}}{.18 \text{ m}} = -66.7 \text{ V/m} \quad E_y \text{ is down}$$

b) parallel plates in the gap between

$$E = \frac{\sigma}{\epsilon_0} \quad E_1 + E_2 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

$$\begin{aligned} \sigma = E\epsilon_0 &= 66.7 \frac{\text{V}}{\text{m}} \cdot 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2 \text{V}} \\ &= 5.9 \times 10^{-10} \frac{\text{C}}{\text{m}^2} \end{aligned}$$

6) A charge of -1.0×10^{-6} C is located at $x = -0.20$ m and a charge of 2.0×10^{-6} C is located at $x = 0.20$ m. Determine the electric field at $(-0.20$ m, 0.20 m) in component form. (Make sure you include a diagram showing the electric field from the individual charges and the total electric field).



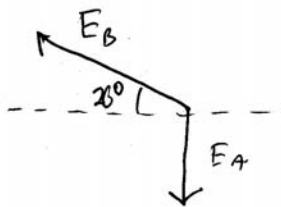
$$E_A = k \frac{q}{r^2} = \frac{8.99 \times 10^9 \frac{Nm^2}{C^2} (1 \times 10^{-6} C)}{(0.2 \text{ m})^2}$$

$$E_A = 224750 \text{ N/C}$$

$$\theta = \tan^{-1}\left(\frac{0.2}{0.4}\right) = 26.6^\circ$$

$$E_B = k \frac{q}{r^2} = \frac{8.99 \times 10^9 \frac{Nm^2}{C^2} (2 \times 10^{-6} C)}{(0.4^2 + 0.2^2)}$$

$$E_B = 89900 \text{ N/C} \quad 74144$$



$$E_{Bx} = E_B \cos \theta = 89900 \text{ N/C} \cos(26) = 80409 \text{ N}$$

$$E_{By} = E_B \sin \theta = 39410 \text{ N/C} \quad 32503$$

$$40256$$

$$\vec{E} = E_{Bx} \hat{i} + (E_{By} - E_A) \hat{j}$$

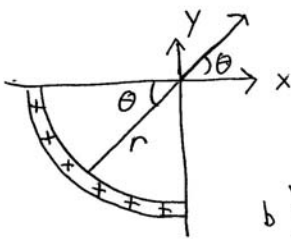
$$\vec{E} = 80409 \hat{i} + (39410 - 224750) \hat{j}$$

$$\vec{E} = -80409 \hat{i} - 185340 \hat{j}$$

$$66640 \hat{i} - 192247 \hat{j}$$

7) A plastic rod of charge $q = 28 \mu\text{C}$ is bent into a quarter circle of radius $r = .2 \text{ m}$ as shown in the figure.

- Assuming the charge is uniformly distributed over the quarter circle, determine the linear charge density λ .
- Write an expression for the x component of the electric field at the origin due to a small piece of charge dq at angle θ .
- Integrate the expression from b to determine the total x component of the electric field.
- Using symmetry arguments write the y component of the electric field.
- Write the total vector electric field in component form.



$$\lambda = \frac{q}{\frac{1}{4} 2\pi r} = \frac{28 \times 10^{-6} \text{ C}}{\frac{\pi}{2} (.2 \text{ m})} = 8.9 \times 10^{-5} \frac{\text{C}}{\text{m}}$$

$$b) \quad dE_x = dE \cdot \cos \theta = \frac{k dq}{r^2} \cos \theta$$

$$c) \quad dq = \lambda r d\theta$$

$$E_x = \int \frac{k dq}{r^2} \cos \theta = \int \frac{k \lambda r d\theta}{r^2} \cos \theta$$

$$E_x = \int_0^{\pi/2} \frac{k \lambda}{r} \cos \theta \cdot r d\theta$$

$$E_x = \frac{k \lambda}{r} \int_0^{\pi/2} \cos \theta \cdot r d\theta$$

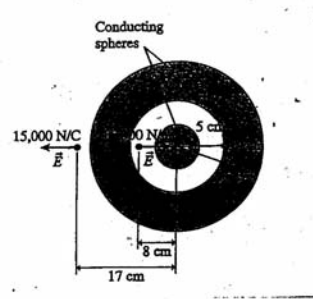
$$E_x = \frac{k \lambda}{r} \sin \theta \Big|_0^{\pi/2} = \frac{k \lambda}{r} =$$

$$E_x = \frac{8.99 \times 10^9 \cdot (8.9 \times 10^{-5})}{.2} = 4 \times 10^6 \text{ N/C}$$

$$\vec{E} = E_x \hat{i} + E_y \hat{j} = 4 \times 10^6 \hat{i} + 4 \times 10^6 \hat{j}$$

8) The figure shows a solid metal sphere at the center of a hollow metal sphere. The electric field is given at a location between the spheres and a location external to the spheres.

- Determine the electric flux (Φ_E) on a spherical Gaussian surface that is concentric with the metal spheres and has a radius 8 cm.
- What is the total charge on the exterior of the inner sphere?
- What is the total charge on both spheres?
- What is the total charge on the inside surface of the hollow sphere?



$$i) \oint \vec{E} \cdot d\vec{a} = EA = 15000 \text{ N/C} \cdot 4\pi(0.08)^2 = 1206.4 \frac{\text{N}\cdot\text{m}^2}{\text{C}}$$

$$b) q_{in} = \epsilon_0 \Phi_E = 8.85 \times 10^{-12} (1206.4) = -1.1 \times 10^{-8} \text{ C}$$

$$c) q_{in} = \epsilon_0 \int \vec{E} \cdot d\vec{a}$$

$$q_{in} = 8.85 \times 10^{-12} (15,000 \text{ N/C}) (4\pi(0.17)^2) = 4.8 \times 10^{-8} \text{ C}$$

d) inside sphere conductor $\vec{E} = 0$

$$\oint \vec{E} \cdot d\vec{a} = \frac{q_{in}}{\epsilon_0} = \frac{q_A + q_B}{\epsilon_0}$$

$$q_A = -1.1 \times 10^{-8} \text{ C}$$

$$q_B = -q_A = 1.1 \times 10^{-8} \text{ C}$$