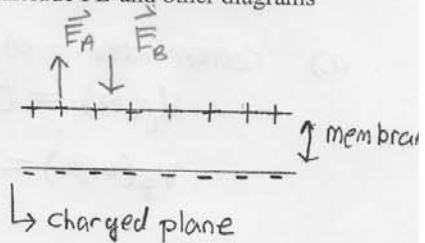


Questions and Problems: Provide clear and logical answers to each of the following questions. Where calculations are required, neatly show all work. You must clearly show all work to receive full credit. Be sure that your answers have the correct units. If you continue your work on another sheet of paper, be sure that it is clearly labeled. Be sure to include FB and other diagrams where appropriate.

1 (20 points) Suppose the plasma membrane of a cell has a surface charge density of about 10 C/m^2 on one side and an equal but opposite surface charge density on the other side.

- Determine the electric field strength produced by this charge distribution inside and outside the plasma membrane.
- Calculate the force on an ion of Ca^{2+} ($q = 2e$) placed inside the plasma membrane of a cell.



$$a) \quad \vec{E}_{in} = -E_A - E_B \hat{j} = -\frac{|\sigma|}{2\epsilon_0} - \frac{|\sigma|}{2\epsilon_0} \hat{j}$$

$$\vec{E}_{in} = -\frac{\sigma}{\epsilon_0} \hat{j} = -\frac{10 \text{ C/m}^2}{8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2} = -1.13 \times 10^{12} \frac{\text{N}}{\text{C}} \hat{j}$$

$\vec{E}_{out} = 0$ The fields produced by planes are oppositely directed and equal in magnitude.

$$b) \Rightarrow \vec{F} = q\vec{E} = 2(1.6 \times 10^{-19} \text{ C}) (1.13 \times 10^{12} \text{ N/C})$$

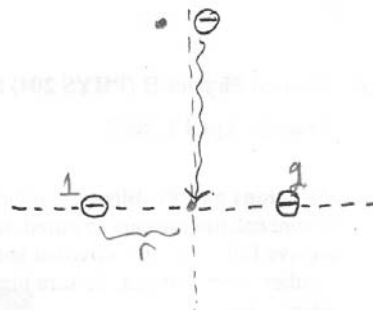
$$= 3.6 \times 10^{-7} \text{ N}$$



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2 (20 points) Two electrons are fixed 2.0 cm apart. Another electron is shot from infinity and stops midway between the two electrons.

- a) Determine the initial speed of the electron?
 b) Determine the energy stored in the charge configuration when the third electron is midway between the two electrons.



a) Conservation of Energy.

$$V_i(\infty) = 0$$

$$V_f(\text{origin}) = k_e \frac{q_1}{r_1} + k_e \frac{q_2}{r_2} = 2k_e \frac{q}{r}$$

$$q = 1.6 \times 10^{-19} \text{ C} \quad r = .01 \text{ m}$$

$$V_f = 2 \cdot 8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{1.6 \times 10^{-19} \text{ C}}{.01 \text{ m}} = 2.88 \times 10^{-7} \text{ V}$$

$$KE_i = ?$$

$$KE_f = 0$$

$$KE_i + qV_i = KE_f + qV_f$$

$$\frac{1}{2} m v_i^2 = (-e) \left(\frac{2k_e(-e)}{r} \right)$$

$$v_i = \sqrt{\frac{4k_e e^2}{r m}} = \sqrt{\frac{4(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(1.6 \times 10^{-19} \text{ C})^2}{.01 \text{ m} \cdot 9.11 \times 10^{-31} \text{ kg}}}$$

$$v_i = 318 \text{ m/s}$$

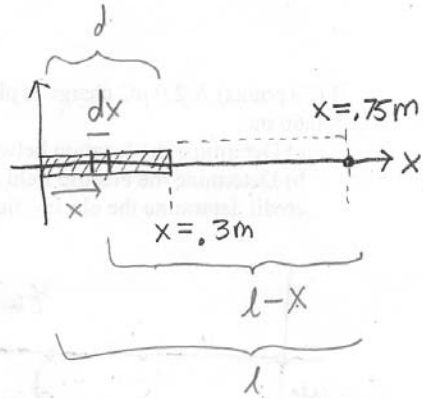
b)
$$U = k_e \frac{q_1 q_2}{r_{12}} + \left[k_e \frac{q_3 q_1}{r_{13}} + k_e \frac{q_2 q_3}{r_{23}} \right] \quad \begin{aligned} q_1 = q_2 = q_3 \\ r_{13} = r_{23} = r \\ r_{12} = 2r \end{aligned}$$

$$U = k_e q^2 \left[\frac{1}{2r} + \frac{2}{r} \right]$$

$$U = \frac{k_e q^2}{2r} \cdot 5 = \frac{8.99 \times 10^9 \text{ Nm}^2/\text{C}^2 (-1.6 \times 10^{-19} \text{ C})^2}{2(.01 \text{ m})} = 5.75 \times 10^{-5} \text{ J}$$

3 (20 points) A 0.30 m rod carries a charge of $80 \mu\text{C}$ spread uniformly over its length.

- Determine the linear charge density of the rod.
- Apply Coulomb's law to this continuous charge distribution to determine the electric field along the axis of the rod 0.45 m from the end of the rod.



$$a) \lambda = \frac{Q}{l} = \frac{80 \mu\text{C}}{0.3 \text{ m}} = 267 \mu\text{C/m}$$

$$b) dE = k_e \frac{dq}{r^2} = k_e \frac{\lambda dx}{(l-x)^2}$$

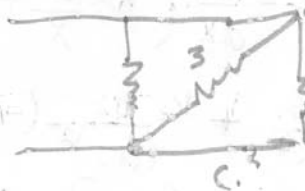
$$E = \int_0^d k_e \frac{\lambda dx}{(l-x)^2} = k_e \lambda \int_0^d \frac{dx}{(l-x)^2}$$

$$(l-x) = u \quad -dx = du$$

$$E = k_e \lambda \int \frac{du}{(u^2)} = -k_e \lambda \frac{u^{-1}}{-1} = k_e \lambda \frac{1}{l-x} \Big|_0^d$$

$$E = k_e \lambda \left[\frac{1}{l-d} - \frac{1}{l} \right]$$

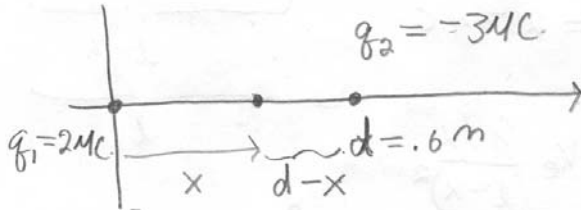
$$E = 8.99 \times 10^9 (267 \times 10^{-6}) \left[\frac{1}{0.75-0.3} - \frac{1}{0.75} \right] = 213 \times 10^6$$



$$3 \parallel 2 = 1.2 \Omega$$

4 (20 points) A $2.0 \mu\text{C}$ charge is placed at the origin. A $-3.0 \mu\text{C}$ charge is located on the x axis at $x = 0.60 \text{ m}$.

- Determine the location between the charges where the electric potential is zero.
- Determine the electric field at the location where the potential is equal to zero (For full credit determine the electric field from the potential, do not use Coulomb's law.)



$$V = k_e \frac{q_1}{x} + k_e \frac{q_2}{(d-x)} = 0$$

$$\frac{q_1}{x} = -\frac{q_2}{d-x}$$

$$q_1 d - q_1 x = -q_2 x$$

$$\frac{q_1 d}{(q_1 - q_2)} = x = \frac{2 \mu\text{C}}{(2 \mu\text{C} - (-3 \mu\text{C}))} (0.6 \text{ m})$$

$$x = 0.24 \text{ m}$$

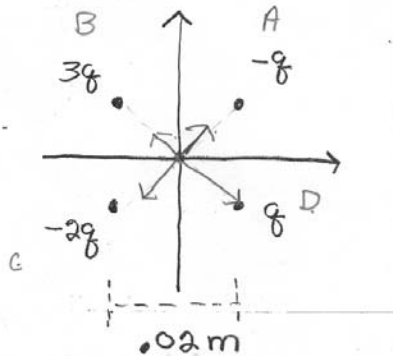
$$E_x = -\frac{dV}{dx} = -\frac{d}{dx} \left(k_e \frac{q_1}{x} + k_e \frac{q_2}{(d-x)} \right)$$

$$E_x = -\frac{k_e q_1}{x^2} - k_e \frac{q_2}{(d-x)^2} (-1)$$

$$E_x = k_e \left[\frac{q_1}{x^2} - \frac{q_2}{(d-x)^2} \right] = 9 \times 10^9 \left[\frac{2 \times 10^{-6}}{0.24^2} - \frac{(-3 \times 10^{-6})}{(0.36)^2} \right]$$

$$= 2203 \times 10^3 = 2.203 \times 10^6$$

5 (20 points) Four charges ($q = 1.0 \times 10^{-8} \text{ C}$) are located at the corners of a square as indicated in the figure. Determine the electric field (in component form) at the center of the square.

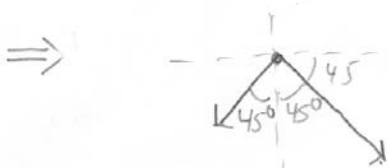


\Rightarrow Field produced by A and C are along the same line can add directly. The magnitude will be.

$$E_{AC} = k_e \frac{|q|}{r^2} \text{ pointing towards C.}$$

Field produced by B & D is along the same line but $3q$ dominates so net field will point towards D

$$E_{BD} = k_e \frac{|2q|}{r^2} = \frac{2q}{r^2} = \frac{2(1.0 \times 10^{-8})}{(0.01^2 + 0.01^2)} = 2 \times 10^{-4}$$



$$E_{AC} = 8.99 \times 10^9 \frac{q(1 \times 10^{-8} \text{ C})}{2 \times 10^{-4}} = 450000 \text{ N/C}$$

$$E_{BD} = 900000$$

$$E_{ACx} = E_{AC} \cos 45 = 450,000 \text{ N/C} \cdot \cos 45 = -318000 \text{ left}$$

$$E_{ACy} = -318000 \text{ down}$$

$$E_{BDx} = +636000 \text{ right}$$

$$E_{BDy} = 636000 \text{ down}$$

$$\vec{E} = (E_{ACx} + E_{BDx}) \hat{i} + (E_{ACy} + E_{BDy}) \hat{j}$$

$$\vec{E} = 318,000 \hat{i} + -954,000 \hat{j}$$