

Exam 3: April 21, 2008

Questions and Problems: Provide clear and logical answers to each of the following questions. You must clearly show all work to receive full credit. Be sure that your answers have the correct units. If you continue your work on another sheet of paper, be sure that it is clearly labeled. Be sure to include FB and other diagrams where appropriate.

1 (20 points) Determine the capacitance of a parallel plate capacitor consisting of square plates (sides of length 0.20 m) separated by 0.0015 m. The plates are maintained at a potential difference of 6.0 V (see figure 1).

- Determine the charge on the capacitor.
- Determine the energy stored in the capacitor.

The plates remain connected to the battery and then the gap between the plates is filled with polystyrene ($\kappa=2.6$).

- Determine the electric potential difference between the plates and the charge stored on the plates.
- Determine the energy stored in the capacitor.

$$C_0 = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} (0.2 \text{ m})^2}{0.0015 \text{ m}} = 2.36 \times 10^{-10} \text{ F}$$

$$a) Q = C \Delta V = 2.36 \times 10^{-10} \text{ F} \cdot 6 \text{ V} = 1.4 \times 10^{-9} \text{ C}$$

$$b) U = \frac{1}{2} C \Delta V^2 = \frac{1}{2} (2.36 \times 10^{-10} \text{ F}) (6 \text{ V})^2 = 4.25 \times 10^{-9} \text{ J}$$

c) still connected to the battery so $\Delta V = 6 \text{ V}$

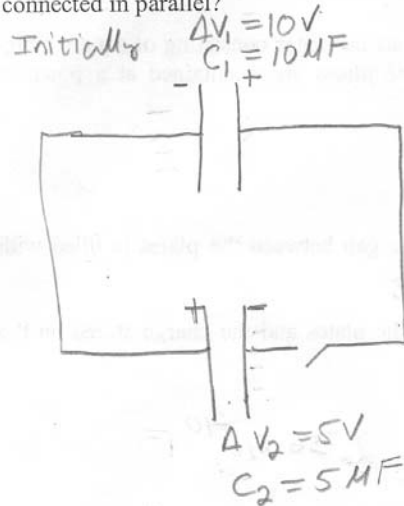
$$Q = C \Delta V = \kappa C_0 \Delta V = 2.36 \times 10^{-10} \text{ F} (2.6) 6 \text{ V}$$

$$Q = 3.68 \times 10^{-9} \text{ C}$$

$$U = \frac{1}{2} Q \Delta V = \frac{1}{2} \cdot 3.68 \times 10^{-9} \text{ C} \cdot 6 \text{ V} = 1.1 \times 10^{-8} \text{ J}$$

2 (20 points) A $10.0\text{-}\mu\text{F}$ capacitor is charged so that the potential difference between its plates is 10.0 V . A $5.0\text{-}\mu\text{F}$ capacitor is similarly charged so that the potential difference between its plates is 5.0 V . The two charged capacitors are then connected to each other in parallel with positive plate connected to positive plate and negative plate connected to negative plate.

- a) How much charge flows from one capacitor to the other when the capacitors are connected?
 b) What is the final potential difference across the plates of the capacitors when they are connected in parallel?



$$a) Q_{1i} = C_1 \Delta V_{1i} = 10^{-4}\text{ C}$$

$$Q_{2i} = C_2 \Delta V_{2i} = 2.5 \times 10^{-5}\text{ C}$$

$$Q_{Ti} = 12.5 \times 10^{-5}\text{ C}$$

$$\Rightarrow Q_{Tf} = 7.5 \times 10^{-5}\text{ C}$$

To be shared

$$\Delta V_{1f} = \Delta V_{2f}$$

$$\frac{Q_{1f}}{C_1} = \frac{Q_{2f}}{C_2}$$

$$Q_{Tf} = Q_{1f} + Q_{2f}$$

$$\frac{Q_{2f}}{Q_{1f}} = \frac{C_1}{C_2} = \text{Charges must be in a 2 to 1 ratio}$$

$$\frac{Q_{Tf} - Q_{2f}}{Q_{2f}} = \frac{C_1}{C_2}$$

$$Q_{Tf} C_2 - Q_{2f} C_2 = C_1 Q_{2f}$$

$$Q_{2f} = Q_{Tf} \frac{C_2}{C_1 + C_2}$$

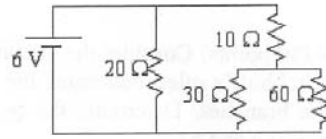
$$\Delta V_{1f} = \frac{Q_{1f}}{C} = \frac{5 \times 10^{-5}\text{ C}}{10 \times 10^{-6}\text{ F}} = 5\text{V}$$

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$$Q_{2f} = 7.5 \times 10^{-5}\text{ C} \frac{5}{15} = 2.5 \times 10^{-5}\text{ C}$$

$$\therefore Q_{1f} = 5 \times 10^{-5}\text{ C}$$

3 (20 points) Consider the circuit shown to the right.



- Determine the equivalent resistance of the circuit.
- Determine the current drawn from the battery.
- Determine the potential difference across the 30 Ω resistor.
- What would happen to the current flow through the 20 Ω resistor if a wire were connected from points A and B?

$$a) \frac{1}{R_{36}} = \frac{1}{30} + \frac{1}{60} \Rightarrow R_{36} = 20\Omega \quad R_{361} = 20\Omega + 10\Omega = 30\Omega$$

$$\frac{1}{R_T} = \frac{1}{20} + \frac{1}{30} = \frac{3}{60} + \frac{2}{60} = \frac{5}{60} \quad R_T = 12\Omega$$

$$b) I = \frac{4V_B}{R_T} = \frac{6V}{12\Omega} = .5 \text{ Amps}$$

$$c) 6V \text{ is across } 20\Omega \text{ and } R_{361} = 30\Omega \text{ so the current flowing through the } 10\Omega \quad I = \frac{4V}{R_{361}} = \frac{6V}{30\Omega} = .2A_s$$

$$\left\{ I = .5A - .2A = \underline{\underline{.3A}} \text{ flows through the } 20\Omega \right\}$$

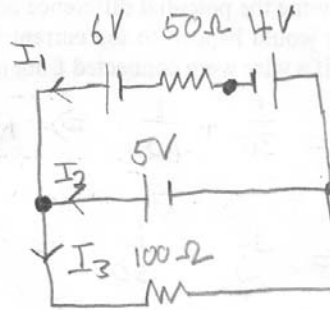
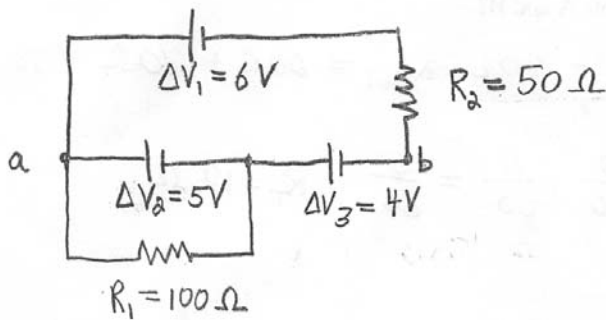
$$AV_{10} = .2A \cdot 10\Omega = 2V \quad \text{so } 4V \text{ is across the } 30\Omega \text{ \& } 60\Omega \text{ pair.}$$

$$d) \frac{1}{R_T} = \frac{1}{20} + \frac{1}{10} \quad R_T = \frac{20}{3} = 6.67\Omega \quad \text{more current will flow overall}$$

$$I = \frac{6V}{6.67\Omega} = .9A \quad \approx \frac{1}{3} \text{ will flow through the } 20\Omega$$

$$\text{which is } \underline{\underline{.3A}}$$

4 (20 points) Consider the circuit shown to the right. Using Kirchhoff's rules, determine the current flow in all three of the branches. Determine the potential difference between points a and b.



$$I_1 + I_2 = I_3$$

$$6V - 5V - 4V - I_1(50\Omega) = 0$$

$$-3V - I_1 \cdot 50 =$$

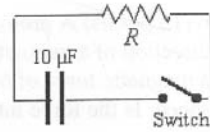
$$I_1 = \frac{-3V}{50\Omega} = -.06 \text{ A flowing into 6V battery.}$$

$$5V - 100 I_3 = 0$$

$$I_3 = \frac{5}{100} = .05 \text{ A}$$

$$I_2 = I_3 - I_1 = .05 \text{ A} - (-.06 \text{ A}) = .11 \text{ A}$$

5 (20 points) The figure to the right shows a simple RC circuit consisting of a $10\text{-}\mu\text{F}$ capacitor in series with a resistor. Initially, the capacitor has been charged and the switch is open as suggested in the figure. The potential difference between its plates is 100 V . At $t = 0\text{ s}$, the switch is closed. The capacitor discharges exponentially so that 2.0 s after the switch is closed, the potential difference between the capacitor plates is 37 V .



- What is the initial charge on capacitor?
- What is the time constant of the circuit?
- Determine the numerical value of the resistance R .
- Find the current flowing in the circuit at 2.0 s .

$$a) \quad C = \frac{Q}{\Delta V} \quad Q = C \Delta V = 10 \times 10^{-6} \text{ F} (100 \text{ V}) = 10^{-3} \text{ C}$$

b) τ is the time to reduce to $q = \frac{q_0}{e} \approx 0.37$ of its original value. That time is $\tau = 2.0\text{ s}$ given above.

$$\text{or } q = q_0 e^{-t/\tau} \quad q_0 = 10^{-3} \text{ C}$$

$$q(2) = 10 \times 10^{-6} \text{ F} (37 \text{ V}) = 3.7 \times 10^{-4} \text{ C}$$

$$\ln q/q_0 = -t/\tau$$

$$\tau = -t / \ln(q/q_0) = -2 / \ln\left(\frac{3.7 \times 10^{-4}}{10^{-3}}\right) = +2.01 \text{ s}$$

$$c) \quad R = \tau / C = \frac{2 \text{ s}}{10 \times 10^{-6} \text{ F}} = 2 \times 10^5 \Omega$$

$$d) \quad I = \frac{dq}{dt} = \frac{d}{dt} (q_0 e^{-t/\tau}) = -\frac{q_0}{\tau} e^{-t/\tau}$$

$$= \frac{-10^{-3} \text{ C}}{2 \text{ s}} e^{-2/2} = 1.84 \times 10^{-4} \text{ A}$$