Q12.7 No; Kinetic, Yes; Potential, No. For constant amplitude, the total energy $\frac{1}{2}kA^2$ stays constant. The kinetic energy $\frac{1}{2}mv^2$ would increase for larger mass if the speed were constant, but here the greater mass causes a decrease in frequency and in the average and maximum speed, so that the kinetic and potential energies at every point are unchanged.

P12.2 (a) $x=(5.00\text{ cm})\cos\left(2t+\frac{\pi}{6}\right)$ At $t=0$, $x=(5.00\text{ cm})\cos\left(\frac{\pi}{6}\right)=\boxed{4.33\text{ cm}}$

(b) $v=\frac{dx}{dt}=-(10.0\text{ cm/s})\sin\left(2t+\frac{\pi}{6}\right)$ At $t=0$, $v=\boxed{-5.00\text{ cm/s}}$

(c) $a=\frac{dv}{dt}=-\left(20.0\text{ cm/s}^2\right)\cos\left(2t+\frac{\pi}{6}\right)$ At $t=0$, $a=\boxed{-17.3\text{ cm/s}^2}$

(d) $A=\boxed{5.00\text{ cm}}$ and $T=\frac{2\pi}{\omega}=\frac{2\pi}{2}=\boxed{3.14\text{ s}}$

P12.5 (a) At $t=0$, $x=0$ and v is positive (to the right). Therefore, this situation corresponds to $x=A\sin\omega t$

and $v=v_i\cos\omega t$

Since $f=1.50\text{ Hz}$, $\omega=2\pi f=3.00\pi$

Also, $A=2.00$ cm, so that $\boxed{x=(2.00\text{ cm})\sin 3.00\pi t}$

(b) $v_{\max}=v_i=A\omega=2.00(3.00\pi)=6.00\pi\text{ cm/s}=\boxed{18.8\text{ cm/s}}$

The particle has this speed at $t=0$ and next at $t=\frac{T}{2}=\boxed{\frac{1}{3}\text{ s}}$

(c) $a_{\max}=A\omega^2=2.00(3.00\pi)^2=18.0\pi^2\text{ cm/s}^2=\boxed{178\text{ cm/s}^2}$

This positive value of acceleration first occurs at $t=\frac{3}{4}T=\boxed{0.500\text{ s}}$

(d) Since $T=\frac{2}{3}$ s and $A=2.00$ cm, the particle will travel 8.00 cm in this time.

Hence, in 1.00 s$\left(=\frac{3}{2}T\right)$, the particle will travel $8.00\text{ cm}+4.00\text{ cm}=\boxed{12.0\text{ cm}}$.

P12.9 $f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ or $T=\frac{1}{f}=2\pi\sqrt{\frac{m}{k}}$

Solving for k, $k=\frac{4\pi^2m}{T^2}=\frac{4\pi^2(7.00\text{ kg})}{(2.60\text{ s})^2}=\boxed{40.9\text{ N/m}}$.

P12.13 Choose the car with its shock-absorbing bumper as the system; by conservation of energy,

$\frac{1}{2}mv^2=\frac{1}{2}kx^2$: $v=x\sqrt{\frac{k}{m}}=\left(3.16\times10^{-2}\text{ m}\right)\sqrt{\frac{5.00\times10^6}{10^3}}=\boxed{2.23\text{ m/s}}$

P12.15 (a) Energy is conserved for the block-spring system between the maximum-displacement and the half-maximum points:

$$(K+U)_i=(K+U)_f \qquad 0+\frac{1}{2}kA^2=\frac{1}{2}mv^2+\frac{1}{2}kx^2$$

$$\frac{1}{2}(6.50\ \text{N/m})(0.100\ \text{m})^2=\frac{1}{2}m(0.300\ \text{m/s})^2+\frac{1}{2}(6.50\ \text{N/m})\left(5.00\times10^{-2}\ \text{m}\right)^2$$

$$32.5\ \text{mJ}=\frac{1}{2}m(0.300\ \text{m/s})^2+8.12\ \text{mJ} \qquad m=\frac{2(24.4\ \text{mJ})}{9.0\times10^{-2}\ \text{m}^2/\text{s}^2}=\boxed{0.542\ \text{kg}}$$

(b) $$\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{6.50\ \text{N/m}}{0.542\ \text{kg}}}=3.46\ \text{rad/s} \qquad \therefore T=\frac{2\pi}{\omega}=\frac{2\pi\,\text{rad}}{3.46\ \text{rad/s}}=\boxed{1.81\ \text{s}}$$

(c) $$a_{\max}=A\omega^2=0.100\ \text{m}(3.46\ \text{rad/s})^2=\boxed{1.20\ \text{m/s}^2}$$

P12.17 (a) $$E=\frac{1}{2}kA^2=\frac{1}{2}(35.0\ \text{N/m})\left(4.00\times10^{-2}\ \text{m}\right)^2=\boxed{28.0\ \text{mJ}}$$

(b) $$|v|=\omega\sqrt{A^2-x^2}=\sqrt{\frac{k}{m}}\sqrt{A^2-x^2}$$

$$|v|=\sqrt{\frac{35.0}{50.0\times10^{-3}}}\sqrt{\left(4.00\times10^{-2}\right)^2-\left(1.00\times10^{-2}\right)^2}=\boxed{1.02\ \text{m/s}}$$

(c) $$\frac{1}{2}mv^2=\frac{1}{2}kA^2-\frac{1}{2}kx^2=\frac{1}{2}(35.0)\left[\left(4.00\times10^{-2}\right)^2-\left(3.00\times10^{-2}\right)^2\right]=\boxed{12.2\ \text{mJ}}$$

(d) $$\frac{1}{2}kx^2=E-\frac{1}{2}mv^2=\boxed{15.8\ \text{mJ}}$$

P12.7 The proposed solution $$x(t)=x_i\cos\omega t+\left(\frac{v_i}{\omega}\right)\sin\omega t$$

implies velocity $$v=\frac{dx}{dt}=-x_i\omega\sin\omega t+v_i\cos\omega t$$

and acceleration $$a=\frac{dv}{dt}=-x_i\omega^2\cos\omega t-v_i\omega\sin\omega t=-\omega^2\left(x_i\cos\omega t+\left(\frac{v_i}{\omega}\right)\sin\omega t\right)=-\omega^2 x$$

(a) The acceleration being a negative constant times position means we do have SHM, and its angular frequency is ω. At $t=0$ the equations reduce to $x=x_i$ and $v=v_i$ so they satisfy all the requirements.

(b) $$v^2-ax=\left(-x_i\omega\sin\omega t+v_i\cos\omega t\right)^2-\left(-x_i\omega^2\cos\omega t-v_i\sin\omega t\right)\left(x_i\cos\omega t+\left(\frac{v_i}{\omega}\right)\sin\omega t\right)$$

$$\begin{aligned}v^2-ax&=x_i^2\omega^2\sin^2\omega t-2x_iv_i\omega\sin\omega t\cos\omega t+v_i^2\cos^2\omega t\\&\quad+x_i^2\omega^2\cos^2\omega t+x_iv_i\omega\cos\omega t\sin\omega t+x_iv_i\omega\sin\omega t\cos\omega t+v_i^2\sin^2\omega t=x_i^2\omega^2+v_i^2\end{aligned}$$

So this expression is constant in time. On one hand, it must keep its original value $v_i^2-a_ix_i$. On the other hand, if we evaluate it at a turning point where $v=0$ and $x=A$, it is $A^2\omega^2+0^2=A^2\omega^2$. Thus it is proved.

***P12.20** (a)

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$-11\text{ m} = 0 + 0 + \frac{1}{2}\left(-9.8\text{ m/s}^2\right)t^2$$

$$t = \sqrt{\frac{22\text{ m}\cdot\text{s}^2}{9.8\text{ m}}} = \boxed{1.50\text{ s}}$$

(b) Take the initial point where she steps off the bridge and the final point at the bottom of her motion.

$$\left(K + U_g + U_s\right)_i = \left(K + U_g + U_s\right)_f$$

$$0 + mgy + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$65\text{ kg } 9.8\text{ m/s}^2\ 36\text{ m} = \frac{1}{2}k(25\text{ m})^2$$

$$k = \boxed{73.4\text{ N/m}}$$

continued on next page

(c) The spring extension at equilibrium is $x = \frac{F}{k} = \frac{65\text{ kg } 9.8\text{ m/s}^2}{73.4\text{ N/m}} = 8.68\text{ m}$, so this point is $11 + 8.68\text{ m} = \boxed{19.7\text{ m below the bridge}}$ and the amplitude of her oscillation is $36 - 19.7 = 16.3\text{ m}$.

(d) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{73.4\text{ N/m}}{65\text{ kg}}} = \boxed{1.06\text{ rad/s}}$

(e) Take the phase as zero at maximum downward extension. We find what the phase was 25 m higher, where $x = -8.68\text{ m}$:

In $x = A\cos\omega t$, $\qquad 16.3\text{ m} = 16.3\text{ m}\cos 0$

$-8.68\text{ m} = 16.3\text{ m}\cos\left(1.06\ \frac{t}{\text{s}}\right) \qquad 1.06\frac{t}{\text{s}} = -122° = -2.13\text{ rad}$

$t = -2.01\text{ s}$

Then $\boxed{+2.01\text{ s}}$ is the time over which the spring stretches.

(f) total time $= 1.50\text{ s} + 2.01\text{ s} = \boxed{3.50\text{ s}}$