Physics 204 Spring 2007 Solutions for homework 2 and 3

P13.1 Replace x by $x - vt = x - 4.5t$

to get
$$\boxed{y = \frac{6}{\left[(x-4.5t)^2+3\right]}}$$

P13.2

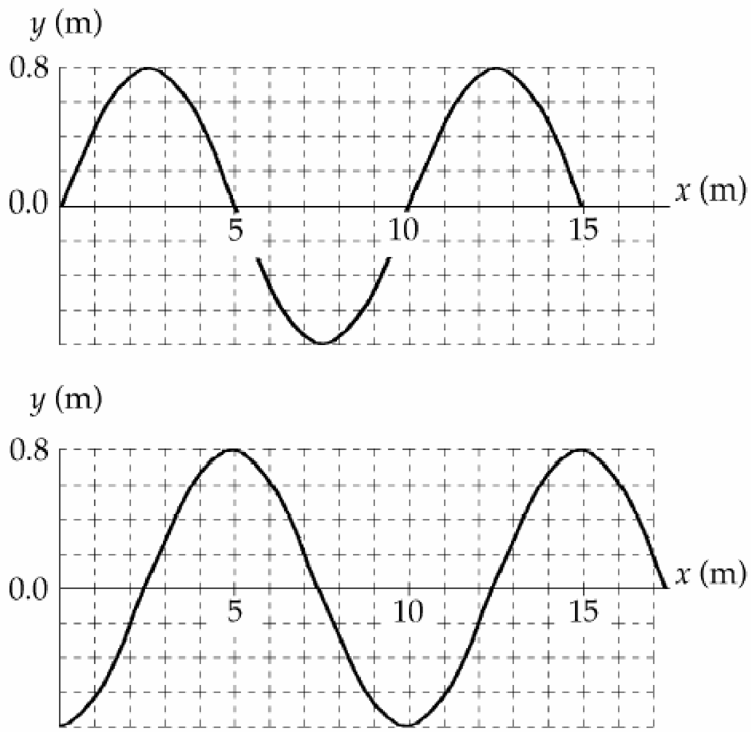

FIG. P13.2

P13.3 $f = \frac{40.0 \text{ vibrations}}{30.0 \text{ s}} = \frac{4}{3} \text{ Hz}$ $\qquad v = \frac{425 \text{ cm}}{10.0 \text{ s}} = 42.5 \text{ cm/s}$

$$\lambda = \frac{v}{f} = \frac{42.5 \text{ cm/s}}{\frac{4}{3} \text{ Hz}} = 31.9 \text{ cm} = \boxed{0.319 \text{ m}}$$

P13.7 (a) $\omega = 2\pi f = 2\pi\left(5\ \text{s}^{-1}\right) = \boxed{31.4\ \text{rad/s}}$

(b) $\lambda = \dfrac{v}{f} = \dfrac{20\ \text{m/s}}{5\ \text{s}^{-1}} = 4.00\ \text{m}$

$k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{4\ \text{m}} = \boxed{1.57\ \text{rad/m}}$

(c) In $y = A\sin(kx - \omega t + \phi)$ we take $A = 12\ \text{cm}$. At $x = 0$ and $t = 0$ we have $y = (12\ \text{cm})\sin\phi$. To make this fit $y = 0$, we take $\phi = 0$. Then $\boxed{y = (12.0\ \text{cm})\sin\left((1.57\ \text{rad/m})x - (31.4\ \text{rad/s})t\right)}$

(d) The transverse velocity is $\dfrac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t)$. Its maximum magnitude is

$A\omega = 12\ \text{cm}(31.4\ \text{rad/s}) = \boxed{3.77\ \text{m/s}}$

(e) $a_y = \dfrac{\partial v_y}{\partial t} = \dfrac{\partial}{\partial t}\left(-A\omega\cos(kx - \omega t)\right) = -A\omega^2\sin(kx - \omega t)$

The maximum value is $A\omega^2 = (0.12\ \text{m})\left(31.4\ \text{s}^{-1}\right)^2 = \boxed{118\ \text{m/s}^2}$.

P13.13 The down and back distance is $4.00\ \text{m} + 4.00\ \text{m} = 8.00\ \text{m}$.

The speed is then $v = \dfrac{d_{\text{total}}}{t} = \dfrac{4(8.00\ \text{m})}{0.800\ \text{s}} = 40.0\ \text{m/s} = \sqrt{\dfrac{T}{\mu}}$

Now, $\mu = \dfrac{0.200\ \text{kg}}{4.00\ \text{m}} = 5.00 \times 10^{-2}\ \text{kg/m}$

So $T = \mu v^2 = \left(5.00 \times 10^{-2}\ \text{kg/m}\right)\left(40.0\ \text{m/s}\right)^2 = \boxed{80.0\ \text{N}}$

P13.17 The total time is the sum of the two times.

In each wire $t = \dfrac{L}{v} = L\sqrt{\dfrac{\mu}{T}}$

Let A represent the cross-sectional area of one wire. The mass of one wire can be written both as $m = \rho V = \rho AL$ and also as $m = \mu L$.

Then we have $\mu = \rho A = \dfrac{\pi \rho d^2}{4}$

Thus, $t = L\left(\dfrac{\pi \rho d^2}{4T}\right)^{1/2}$

For copper, $t = (20.0)\left[\dfrac{(\pi)(8\,920)\left(1.00 \times 10^{-3}\right)^2}{(4)(150)}\right]^{1/2} = 0.137\ \text{s}$

For steel, $t = (30.0)\left[\dfrac{(\pi)(7\,860)\left(1.00 \times 10^{-3}\right)^2}{(4)(150)}\right]^{1/2} = 0.192\ \text{s}$

The total time is $0.137 + 0.192 = \boxed{0.329\ \text{s}}$

P14.12 $y = (1.50\text{ m})\sin(0.400x)\cos(200t) = 2A_0 \sin kx \cos \omega t$

Therefore, $k = \dfrac{2\pi}{\lambda} = 0.400 \text{ rad/m}$ $\qquad \lambda = \dfrac{2\pi}{0.400 \text{ rad/m}} = \boxed{15.7 \text{ m}}$

and $\omega = 2\pi f$ so $\qquad f = \dfrac{\omega}{2\pi} = \dfrac{200 \text{ rad/s}}{2\pi \text{ rad}} = \boxed{31.8 \text{ Hz}}$

The speed of waves in the medium is $v = \lambda f = \dfrac{\lambda}{2\pi} 2\pi f = \dfrac{\omega}{k} = \dfrac{200 \text{ rad/s}}{0.400 \text{ rad/m}} = \boxed{500 \text{ m/s}}$

P14.17 $L = 30.0 \text{ m}$; $\mu = 9.00 \times 10^{-3} \text{ kg/m}$; $T = 20.0 \text{ N}$; $f_1 = \dfrac{v}{2L}$

where $\qquad v = \left(\dfrac{T}{\mu}\right)^{1/2} = 47.1 \text{ m/s}$

so $\qquad f_1 = \dfrac{47.1}{60.0} = \boxed{0.786 \text{ Hz}} \qquad f_2 = 2f_1 = \boxed{1.57 \text{ Hz}}$

$f_3 = 3f_1 = \boxed{2.36 \text{ Hz}} \qquad f_4 = 4f_1 = \boxed{3.14 \text{ Hz}}$

P12.23 Using the simple harmonic motion model:

$$A = r\theta = 1\text{ m}\; 15°\frac{\pi}{180°} = 0.262\text{ m}$$

$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8\text{ m/s}^2}{1\text{ m}}} = 3.13\text{ rad/s}$$

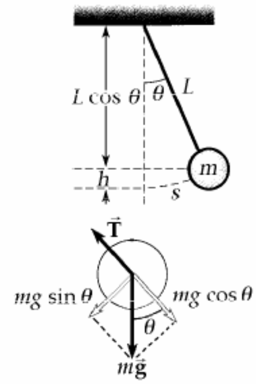

FIG. P12.23

(a) $v_{max} = A\omega = 0.262\text{ m}\; 3.13/\text{s} = \boxed{0.820\text{ m/s}}$

(b) $a_{max} = A\omega^2 = 0.262\text{ m}(3.13/\text{s})^2 = 2.57\text{ m/s}^2$

$a_{tan} = r\alpha \qquad \alpha = \frac{a_{tan}}{r} = \frac{2.57\text{ m/s}^2}{1\text{ m}} = \boxed{2.57\text{ rad/s}^2}$

(c) $F = ma = 0.25\text{ kg}\; 2.57\text{ m/s}^2 = \boxed{0.641\text{ N}}$

More precisely,

(a) $mgh = \frac{1}{2}mv_{max}^2$ and $h = L(1-\cos\theta)$

$\therefore v_{max} = \sqrt{2gL(1-\cos\theta)} = \boxed{0.817\text{ m/s}}$

(b) $I\alpha = mgL\sin\theta$

$\alpha_{max} = \frac{mgL\sin\theta}{mL^2} = \frac{g}{L}\sin\theta_i = \boxed{2.54\text{ rad/s}^2}$

(c) $F_{max} = mg\sin\theta_i = 0.250(9.80)(\sin 15.0°) = \boxed{0.634\text{ N}}$