

Lesson 15

Improper Integrals

Initializations

```
> restart;
```

15.1 Infinity as an Integration Limit

Examples

Example 15.1.1

Evaluate $\int_1^{\infty} \frac{\ln x}{x^3} dx$.

Solution

As explained in class

$$\int_1^{\infty} \frac{\ln x}{x^3} dx = \lim_{a \rightarrow \infty} \int_1^a \frac{\ln x}{x^3} dx$$

We have learned in an earlier section how to evaluate the integral on the right. At this time we simply ask Maple for its value.

```
> e1:=Int(ln(x)/x^3, x=1..a);
```

$$e1 := \int_1^a \frac{\ln(x)}{x^3} dx \quad (2.1.1.1)$$

```
> e2:=value(e1);
```

$$e2 := \frac{1}{4} \frac{a^2 - 2 \ln(a) - 1}{a^2} \quad (2.1.1.2)$$

Finally, we take the limit of this result for $a \rightarrow \infty$.

```
> e3:=Limit(e2, a=infinity);
```

$$e3 := \lim_{a \rightarrow \text{infinity}} \left(\frac{1}{4} \frac{a^2 - 2 \ln(a) - 1}{a^2} \right) \quad (2.1.1.3)$$

```
> ans:=value(e3);
```

$$\text{ans} := \frac{1}{4} \quad (2.1.1.4)$$

15.2 Singularities in the Integrand.

Examples

Example 15.2.1

Evaluate $\int_1^e \frac{1}{x (\ln x)^{1/4}} dx$.

Solution

Clearly, the integrand has a singularity at 1. We replace the lower limit by a and take the limit for $a \rightarrow 1^+$.

$$\int_1^e \frac{1}{x (\ln x)^{1/4}} dx = \lim_{a \rightarrow 1^+} \int_a^e \frac{1}{x (\ln x)^{1/4}} dx$$

```
> e1:=Int(1/(x*ln(x)^(1/4)), x=a..exp(1));
```

$$e1 := \int_a^e \frac{1}{x \ln(x)^{1/4}} dx \quad (3.1.1.1)$$

Let Maple evaluate this integral.

```
> e2:=value(e1);
```

$$e2 := -\frac{4}{3} \ln(a)^{3/4} + \frac{4}{3} \quad (3.1.1.2)$$

Take the limit of this result for $a \rightarrow 1^+$.

```
> e3:=Limit(e2, a=1, right);
```

$$e3 := \lim_{a \rightarrow 1^+} \left(-\frac{4}{3} \ln(a)^{3/4} + \frac{4}{3} \right) \quad (3.1.1.3)$$

```
> ans:=value(e3);
```

$$ans := \frac{4}{3} \quad (3.1.1.4)$$

```
>
```