'Beauty is truth, truth beauty, – that is all
Ye know on earth, and all ye need to know.'

John Keats,
Ode on a Grecian Urn.

Aesthetic Arguments

(Paul Halmos; via Kevin Wood, Oberlin)

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One solution:
$$32 + 16 + 8 + 4 + 2 + 1 = 63$$

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What do these questions have in common?

What if there were 128 teams?

What if two teams had byes until there were only four teams left?

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- This solution is straightforward.
- It tells us *why* the answer is what the answer is.
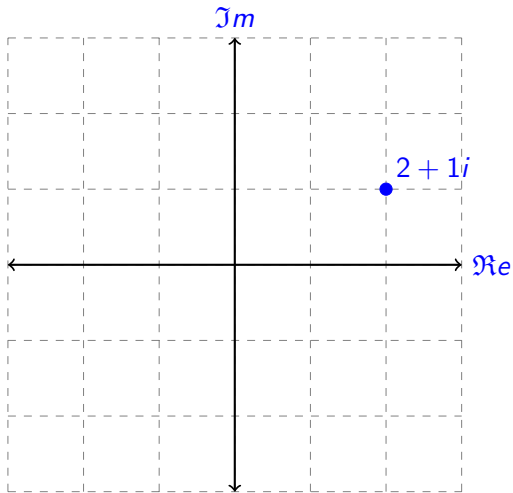- Moreover, it answers a more general question.

Beautiful Results

$$e^{i\pi} + 1 = 0$$

The above true statement combines 5 of the most important numbers in existence into one astounding equation.

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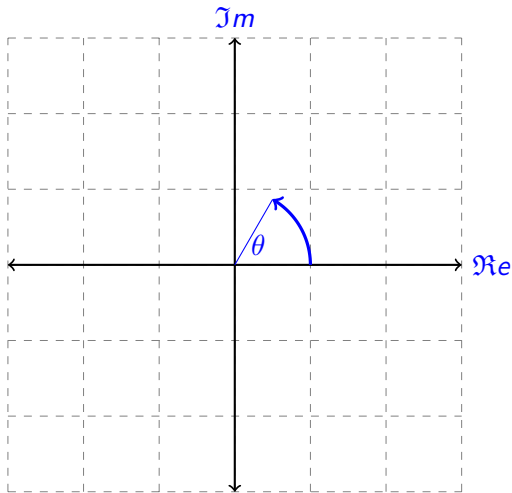(Euler) Imaginary powers of e give the unit circle in the complex plane

$$e^{i\theta} = \cos\theta + i\sin\theta$$

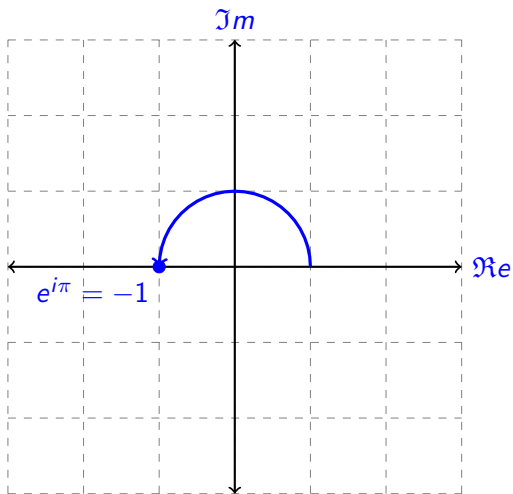where θ is measured in radians.

(Euler) Imaginary powers of e give the unit circle in the complex plane

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π radians equals $180°$.

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Mathematics, **rightly viewed**, *possesses not only truth, but surpreme beauty — a beauty cold and austere...*

... The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the hightest excellence, is found in mathematics as surely as poetry.

Bertrand Russel
Mysticism and Logic: And Other Essays

(Emphasis mine)