Math 173.7 | Fall 08 | Final Exam Review

1. Graph the function $f(x) = x^2 - 3x$ by applying a transformation to the graph of a standard function.

2.a. Express the function $F(x) = 3\ln(x+2)$ in the form $F = f \circ g$, for some functions f and g.

b. Express the function $F(x) = 2^{x-5}$ in the form $f \circ g$.

c. Let $k(x) = x^3$. Give a formula for the function obtained by shifting the graph of k two units up and three units to the right.

d. Find a formula for the inverse of the function $h(x) = \frac{x}{x+1}$. What is the range of h^{-1}? Explain.

3. Find the domain of the given function.

a. $f(x) = \sqrt{4 - 2^x}$

b. $g(x) = \frac{x}{1-e^x}$

c. $f(x) = \sqrt{x-1}$.

d. $g(t) = \frac{\sin(t)}{t}$.

e. $f(x) = \frac{x^3-1}{x^2-5x+4}$

4.a. Find a formula for the inverse of the function $f(x) = \frac{2}{2x+3}$.

b. What is the range of f^{-1}? Explain.

5. Solve the inequality $\ln x + 1 > 0$ for x. Show your work.

6. The point $P = (0, 0)$ lies on the curve $y = e^x - 1$. Let Q be the point $(x, e^x - 1)$.

a. Find a formula for the slope of the secant line PQ.

b. Express the slope of the tangent line to the curve at the point P as a limit.

c. Let P be the point $(0, 1)$ and let Q be the point $(x, \cos x)$.

d. Explain why P and Q lie on the graph of the function $y = \cos x$.

e. Find a formula for the slope of the secant line PQ.

f. Express the slope of the tangent line to the curve at the point P as a limit.

7.a Find a number $\delta > 0$ such that $|x^2 - 4| < 0.1$ whenever $0 < |x - 2| < \delta$.

b. State the precise (that is, ϵ, δ) definition of the limit of a function.

c. Let $f(x) = x^2$. Use the definition to prove that $\lim_{x\to 2} f(x) = 2$.

8. Evaluate the limit, if it exists. Show your work.

a. $\lim_{t\to 3} \frac{t^2+2t-3}{t+3}$

b. $\lim_{x\to 5} \frac{\frac{1}{x}-\frac{1}{5}}{5-x}$

9. Suppose a box is 3 inches high and its length is twice its width. Find a formula for the surface A (in sqaure inches) of the box as a function of its length l (in inches).

10. If a rock is thrown upwards on Mars with a velocity of $10m/s$, its height h in meters t seconds later is given by the following table:

t (seconds)	0	1	2	3	4	5
h (meters)	0	8.14	12.56	13.26	10.24	3.50

a. Find the average velocity between 1 and 4 seconds.

b. Find the average velocity between 0 and 3 seconds.

c. Find the average velocity between 4 and 5 seconds.

d. Express the instantaneous velocity at $t = 2$ as a limit.

e. Express the instataneous velocity at $t = 4$ as a limit.

11. Using the appropriate limit laws, evaluate the limit, if it exists. If it does not exist, explain why. Show your work.

a. $\lim_{t\to 2} \frac{t^2-t-2}{2t-4}$.

b. $\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{3-x}$.

c. $\lim_{x\to 0}(\cos x + \sin x + x^7 + 2)$.

12.a. State the Intermediate Value Theorem.

b. Explain why $f(x) = x^5 + x + 1$ is continuous on the closed interval $[-1, 1]$.

c. Explain why $f(x) = x^2 - e^x$ is continuous on the closed interval $[-1, 0]$.

d. Prove that the equation $x^2 = e^x$ has at least one solution.

e. Prove that the equation $x^5 = -x - 1$ has at least one solution.

13. Give a precise (that is, ϵ, δ) definition of the limit of a function.

14. Let $f(x) = 5x^2 - 1$. Use the definition to prove that $\lim_{x\to 0} f(x) = -1$.

15.a. Let $a \neq -1$ be a number. Express the slope of the tangent line of $y = \frac{x-1}{x+1}$ at the point $(a, \frac{a-1}{a+1})$ as a limit. Do **not** evaluate the limit.

b. Evaluate the limit.

c. Find the equation of the tangent line at the point $(1, 0)$.

d. Let a be a number. Express the slope of the tangent line of $y = x + 2/x$ at the point $(a, a + 2/x)$ as a limit. Do **not** evaluate the limit.

e. Evaluate the limit.

16. Differentiate the function.

a. $f(x) = x^7 + 2x^3 - 2x + 1$

b. $g(t) = t + \frac{1}{t^2+1}$

c. $y = x^2 e^x$

d. $c(\theta) = 1 + \cot\theta$

e. $r(x) = e^{-x} \sin x$

f. $f(x) = 4x^7 + x^3 - x + 1$

g. $g(t) = \frac{t}{t^2+1}$

h. $h(y) = (y + 3)e^y$

17.a. Let a be a number. Find the equation of the tangent line to the curve $y = (2x+5)^3$ at the point $(a, (2a + 5)^3)$.

b. For what values of the number a is the tangent line to the curve $y = (2x + 5)^3$ parallel to the line $y = 54x + 7$?

18. Differentiate the function:

a. $f(x) = \sin(x^3 + 8)$

b. $g(x) = \ln(7x^2 + 5)$

c. $h(t) = \sqrt{\frac{t}{t^2+4}}$

d. $y = e^{ax} \sin bx$

e. $y = 2x \log_{10} \sqrt{x}$

f. $y = x^2 \tan(x - 1) + \ln(x + 1)$.

g. $y = x2^{x^2-1}$.

19. The equation $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ describes a curve in the plane.

a. Verify that the point $(-3, 1)$ lies on the curve.

b. Find the equation of the tangent line to the curve at the point $(-3, 1)$.

20. Differentiate the function $f(x) = (x + 1)^{\sin x}$.

21. Consider the curve $y = x \tan x$.

a. Verify that the point $(\pi/4, \pi/4)$ lies on the curve.

b. Find $\frac{dy}{dx}$.

c. Find the equation of the tangent line to the curve at the point $(\pi/4, \pi/4)$.

22.a. Let a be a number. Find the equation of the tangent line to the curve $y = x^2$ at the point (a, a^2).

b. For what values of the number a does the tangent line to the curve $y = x^2$ pass through the point $(0, 1)$?

23. If a ball begins rolling down a certain hill with initial velocity 3 m/s, then it will roll a distance of $s = 3t + 2t^2$ meters after t seconds.

a. Find the velocity at time t.

b. How long will it take for the ball to be rolling at a velocity of 35 m/s?

24. The population P of a bacteria sample grows at a rate proportional to P; that is $\frac{dP}{dt} = kP$ for some constant k. The initial population is 100 bacteria, and the sample grows to 105% of its original population after one hour.

a. Find the constant k, and give an explicit formula for P as a function of t.

b. How long will it take for the population to double?

25. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4 PM?

26. A spotlight on the ground shines on a wall 12 m away. If a man 1.5 m tall walks from the spotlight towards the building at a speed of 3 m/s, how fast is the length of his shadow on the building decreasing when he is 4 m away from the spotlight?

27. If a spherical snowball melts so that its surface area decreases at a rate of 1 cm/min, find the rate at which the diameter decreases when the diameter is 10 cm.

28. One side of a right triangle is 20 cm long and the adjacent angle is measured to be 45°, with a possible error of ±1°. Use differentials to estimate the error in computing the length of the hypotenuese.

29.a. Find the critical numbers of the function $f(x) = \frac{x^2+1}{x+1}$.

b. Find the global maximum and minimum values of the function $g(x) = \sqrt{x}(4 - x)$ on the interval $[0, 2]$.

30. Let f be continuous and differentiable everywhere. If $f(2) = 12$ and $f'(x) \leq 1$ for all x, what is the smallest possible value of $f(0)$? Justify your answer.

31. Let $f(x) = x\sqrt{x-3}$.

a. Find the intervals on which f is increasing and those on which f is decreasing.

b. Find the intervals of concavity and the inflection points of f.

c. Find the local maxima and minima of f.

32. Find a positive number such that the sum of the number and its reciprocal is as small as possible.

33. A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the amount of fence used?

34. Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 8 - x^2$.

35. Find the largest area of a rectangle that can be inscribed in a right triangle with legs of of lengths 3 cm and 4 cm if two sides of the rectangle lie along the legs.

36. Find the most general antiderivative of the given function.

a. $f(x) = 5x^2 + 4x^3 - 2x$

b. $f(x) = \sqrt{x}$

c. $f(x) = \cos x + 1$

d. $f(x) = e^{-x}$

e. $f(x) = \frac{2}{x} + x$

37. A particle is moving with the given data. Find the position s of the particle as a function of t.

a. $a(t) = 5t + 1$, $v(1) = 2$, $s(0) = 3$.

b. $a(t) = \sin x$, $v(0) = 0$, $s(0) = 0$.

c. $a(t) = 2\sqrt{t}$, $s(1) = 0$, $s(0) = 1$.