PART II

FABRY-PEROT OPTICS

ASSEMBLY: If the Fabry-Perot Optics are ordered with the M-4 Interferometer, unpacking and assembly instructions are the same as for the Michelson Optics (see page 1). If the Fabry-Perot Optics or the Michelson Optics are ordered later, they may be easily installed by removing the masking tape. Then the optical frames can be attached with screws to the drilled and tapped holes.

Under no conditions should the Fabry-Perot mirrors ever be touched with anything but a soft brush—such as the camel’s hair brush, which is included with each order—or lens paper. Any cleaning fluid of any kind will seriously decrease the reflectivity of the mirrors and may possibly necessitate recoating.

LIGHT SOURCE: Both mercury and sodium light sources will be needed for Fabry-Perot experiments. The Atomic Laboratories’ Mercury Light Source is quite satisfactory. For Fabry-Perot experiments it is positioned as shown below.

![Diagram of Fabry-Perot Optics](image)

TELESCOPE: A 6 to 10 power telescope which can focus at infinity is desirable for performance of the following experiments. It is helpful, but not essential, that the telescope be provided with a reticle.


ADJUSTMENT: Loosen the carriage lock screw and adjust the carriage until the Fabry-Perot mirrors are about 1 millimeter apart. Large distances between mirrors makes adjustment more difficult. Now tighten the carriage lock screw.

The next step is to bring the mirror mounted on the base into exact parallelism with the mirror mounted on the carriage. This adjustment can be accomplished as follows.

Turn on the light source and observe multiple reflections in the mirrors. The initial steps in adjustment are accomplished more easily without the telescope in place. A black card having a pinhole in the center placed between the diffuser plate and the Fabry-Perot is sometimes helpful in observing the multiple reflections.

Bring the multiple reflections into coincidence by appropriate turning of the two adjusting screws. The orientation of the base-mounted plate is adjusted about the horizontal axis by the center adjusting screw, and about the vertical axis by the adjusting screw on the right side. If turning of the side adjusting screw is not adequate to bring the multiple images into coincidence, the Allen head screw next to the side adjusting screw may need adjustment.

When coincidence of the multiple images is obtained, remove the black pinhole card. A set of circular fringes should be visible. Precise adjustment for parallelism of the plates is now accomplished. If by moving the eye up and down the circular fringes appear to shrink and expand, a slight adjustment of the center adjusting screw is necessary. If by moving the eye from side to side the circular fringes appear to shrink and expand, a slight adjustment of the side adjusting screw is necessary. These adjustments should be continued until the shrinking and expanding of the circular fringe pattern is minimized and symmetrical about the center.

DISCUSSION: In the Michelson Interferometer, the incoming light is split into two beams, each of which travels a different path before they are brought together to interfere. In the Fabry-Perot interferometer, the incoming light is split into many beams—the multiple reflections observed as the plates were being adjusted. The fringes become sharper as the number of reflections increases. The number of reflections increases as the coating on the mirrors (plates) becomes thicker and less transparent. In this instrument, the number of interfering beams is between 10 and 20, making the width of the fringes about 1/10 to 1/20 the distance between successive fringes. (In the Michelson interferometer, the fringes are just one-half as wide as the distance between fringes.) Wave length measurements are therefore much more accurate when made using a Fabry-Perot interferometer. In most frequent practice, the two mirrors (usually called plates) are separated by fixed spacers in an arrangement called an etalon.

EXPERIMENT 1: Measurement of Sodium Doublet Separation.

Procedure: The sodium doublet consists of two spectral lines in the yellow having wave lengths of 5890 and 5896 Angstrom Units. The 5890 Å line is twice as intense as the 5896 Å line.
Use a sodium light source (for example Cenco Cat. No. 87300) to establish a fringe pattern. When a good fringe pattern has been obtained, turn the micrometer head until the rings due to the weaker line (the less intense rings) are halfway between the brighter rings. Take a reading of the micrometer head. Now turn the micrometer head toward larger reading values until the weaker rings coincide with the stronger ones and then separate until the weaker rings are again halfway between the rings due to the stronger line. Take a new reading of the micrometer head. The formula for the fringe system of a Fabry-Perot Interferometer is:

\[ m\lambda = 2\mu \cos \theta \]

Where:  
\( m \) is the order of interference.  
\( \lambda \) is the wave length of the light.  
\( \mu \) is the index of refraction of the medium between the mirrors.  
\( \theta \) is the separation between the mirrors.  
\( \mu \) is the angle measured from the normal to the mirrors.

With air for the medium between the mirrors, we have \( \mu \approx 1 \). At the center of the fringe pattern \( \cos \theta = 1 \). Our equation becomes:

\[ m\lambda = 2\mu \]

For our first reading we have:

\[ 2\mu_1 = m_1\lambda_1 = (m_1 + n + 1/2) \lambda_2 \]

Where \( \lambda_1 \) is greater than \( \lambda_2 \). The last term on the right-hand side means that the order of the shorter wave length ring system must differ from that of the longer wave length ring system by an odd half integer. This is so because the ring patterns have been adjusted to fall midway between each other.

For our second reading we have:

\[ 2\mu_2 = m_2\lambda_1 = (m_2 + n + 3/2) \lambda_2 \]

(Note that if we had started with the plates in contact with each other, the quantity \( n \) would not have appeared in the two equations immediately preceding.)

By subtraction we obtain:

\[ 2(\mu_2 - \mu_1) = (m_2 - m_1)\lambda_1 = (m_2 - m_1 + 1) \lambda_2 \]

\[ (m_2 - m_1) (\lambda_1 - \lambda_2) = \lambda_2 \]

\[ (m_2 - m_1) = \frac{\lambda_2}{\lambda_1 - \lambda_2} \]

\[ 2(\mu_2 - \mu_1) = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} \]

Since \( \lambda_1 \) and \( \lambda_2 \) are approximately equal, we then obtain:

\[ \lambda_1 - \lambda_2 = \frac{\lambda_2^2}{2(\mu_2 - \mu_1)} \]

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The separation \((t_2 - t_1)\) is evaluated as in the case of the Michelson
interferometer as:

\[
t_2 - t_1 = 0.10 \left( D_2 - D_1 \right) K
\]

Where \((D_2 - D_1)\) is the change of the micrometer reading as read in millimeters,
and \(K\) is the ratio of carriage movement to micrometer reading.

\[
K = 0.020
\]

so:

\[
(t_2 - t_1) = 2.0 \times 10^{-3} \left( D_2 - D_1 \right).
\]

Finally, the doublet separation is given by:

\[
\Delta \lambda = \frac{\lambda^2}{0.4(D_2 - D_1)} \text{ cm.} = \frac{3.47}{(D_2 - D_1)} \text{ ÂÄ}
\]

\[
\Delta \lambda = \frac{\lambda^2}{4 \times 10^{-3}(D_2 - D_1)} \text{ cm.} = \frac{86.8}{(D_2 - D_1)} \text{ ÂÄ}
\]

The procedure just described assumes that the viewing telescope is not fitted
with a reticle. If the telescope is fitted with a reticle, then it is no long-
er essential that the weaker fringe pattern be precisely centered in the strong-
er fringe pattern. This will be discussed in greater detail below.

By use of the method described above, where the fringe pattern due to the
weaker line has been displaced one order with respect to the fringe pattern
due to the stronger line by turning the micrometer head, the following data
were obtained.

<table>
<thead>
<tr>
<th>(D_2) (in.)</th>
<th>(D_1) (in.)</th>
<th>(\lambda_2 - \lambda_1) (ÂÄ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.50</td>
<td>11.18</td>
<td>6.06</td>
</tr>
<tr>
<td>19.90</td>
<td>5.40</td>
<td>5.98</td>
</tr>
<tr>
<td>22.52</td>
<td>8.06</td>
<td>6.00</td>
</tr>
</tbody>
</table>

**Table 1**

Combining these results we obtain an average value for \(\lambda_2 - \lambda_1\) of 6.01 ÂÄ.

When a telescope having a graduated reticle is used, a more precise determi-
nation of the difference in wave length between the members of the sodium dou-
blet may be obtained. A reticle containing 5 to 10 millimeters graduated in
teniths of a millimeter is preferable. A reticle containing 0.2 to 0.4 inches
graduated in 0.005 inches would be adequate.

One side of the fringe pattern is observed by the telescope. This is accom-
plished by moving the telescope so that there is a slight angle between the
axis of the telescope and the normal to the Fabry-Perot plates.
With both weak and strong fringe patterns resolved from each other, and with the fringe patterns as sharp as fine adjustment of the adjusting screws will permit, measure radially from the center of the pattern the position at least some ten fringes, starting several fringes from the center of the pattern. After measuring the fringes, locate the first fringe measured to be sure the fringe pattern has not altered while the measurements were being made. Thermal equilibrium and absence of vibration are necessary for obtaining stable fringe patterns. Read the micrometer. Turn the micrometer head some fifteen to twenty-five revolutions toward higher readings while watching the fringes. The weak and strong fringe patterns will first merge and then again be resolved.

Again measure the position of at least ten fringes outward starting several fringes from the center of the pattern. Check the location of the first fringe measured to determine stability of the pattern. Again read the micrometer.

Analysis of the data is accomplished by use of the "Off-Centre Reduction Method" described on pages 133 and 134 of High Resolution Spectroscopy by S. Tolansky, Pitman Publishing Corporation, 1947.

At the center of a fringe system \( \cos \theta = 1 \) we have \( m \lambda = 2t \). For a circular fringe not in the center of the pattern we have \( m \lambda = 2t \cos \theta \). Thus \( m = m_0 \cos \theta \) where \( \theta \) is the angular radius of a ring of order of interference \( p \). The order of interference of the \( p \)th ring in the system is \( m = m_0 - p \) (with a Fabry-Perot Interferometer the highest order of interference is at the center of the fringe pattern).

To a close approximation \( m = m_0 \cos \theta \) can be written \( m = m_0 - m_0 \frac{\Delta^2}{2} \)

for the small angles under consideration. The angular radius (in radians) for the \( p \)th ring is therefore \( \theta = \sqrt{\frac{2p}{m_0}} \). Thus, in proceeding outward from the center of a fringe pattern the successive circular fringes close up in accordance with a parabolic formula since the squares of the radii are in arithmetic progression.

If

\[
S_2 \Delta_2 = W_2 \Delta_1 W_2 = \Delta_3, \quad S_3 \Delta_3 = \Delta_4, \quad \text{etc.}
\]

\[
W_1 \Delta_1 W_2 = \Delta_2 W_2 = \Delta_3, \quad \text{etc.} \quad \text{(See Figure 2 on page 20)}
\]

\[
S_2 S_3 = \Delta_2 S_3, \quad S_3 S_4 = \Delta_3 S_4, \quad \text{etc.}
\]

then to a reasonably close approximation the fraction of an order \( dm \) between the strong and weak line is given by

\[
dm = \frac{2 \Delta_2}{(\Delta_1 W_2 + \Delta_2 S)} = \frac{2 \Delta_3}{(\Delta_2 W_2 + \Delta_3 S)} = \ldots \text{ for the initial micrometer reading}
\]

Thus a series of independent values of \( dm \) is obtained and sufficient values lead to a good average. A diagram of the appearance of the fringe pattern
and how the $\Delta$'s are obtained appears below.

Fig. 2

In Figure 2 members of the fringe pattern due to the stronger line of the sodium doublet have been designated by the letter $S$, and those due to the weaker line by the letter $W$. From the measurements taken of the fringe pattern at the initial micrometer reading, the $\Delta$s ($\Delta_1$, $\Delta_2$, etc.) will represent the distance from a strong fringe to the next weak fringe outward from the center of the pattern. From the measurements taken at the final micrometer reading, the $\Delta$s will represent the distance from a weak fringe to the next strong fringe outward from the center of the pattern.

The formula for the difference in wave length is:

$$\Delta\lambda = \lambda^2 \frac{dm_f - (-dm_i)}{2d} = \lambda^2 \frac{dm_f + dm_i}{2 \times 2 \times 10^{-3} (D_f - D_i)}$$

where $\Delta\lambda$ and $\lambda$ are in centimeters and $D_f$ and $D_i$ are the final and initial micrometer readings in millimeters. $\lambda$ is approximately $5.89 \times 10^{-5}$ cm.

The initial value of the fractional order $dm_i$ is indicated as negative in the formula above because it has been measured in the opposite sense from the final value, $dm_f$.

An example of data taken and its reduction is given below.

$$D_i = 7.77 \text{ millimeters}$$

\begin{align*}
  w_1 &= 1.50 \\
  s_2 &= 2.15 \\
  w_2 &= 2.55 \\
  s_3 &= 3.15 \\
  w_3 &= 3.50 \\
  s_4 &= 4.05 \\
  w_4 &= 4.35 \\
  s_5 &= 4.85 \\
  \Delta_2 &= 0.40 \\
  \Delta_3 &= 0.35 \\
  \Delta_4 &= 0.30 \\
  \Delta_1W &= 1.05 \\
  \Delta_2S &= 1.00 \\
  \Delta_2W &= 0.95 \\
  \Delta_3S &= 0.90 \\
  \Delta_3W &= 0.85 \\
  \Delta_4S &= 0.80
\end{align*}
\[ \frac{s_1}{w_1} = \frac{1.15}{1.60} = 0.72 \quad \frac{s_2}{w_2} = \frac{2.00}{2.45} = 0.82 \quad \Delta s = 0.85 \]
\[ \Delta_1 = 0.40 \quad \Delta_{ls} = 0.85 \]
\[ \Delta_{lw} = 0.85 \quad \Delta_{2s} = 0.80 \quad \Delta_{2w} = 0.75 \]
\[ \Delta_2 = 0.35 \quad \Delta_{3s} = 0.70 \quad \Delta_{3w} = 0.70 \]
\[ \Delta_3 = 0.30 \]
\[ \frac{\Delta_n}{w_4} = \frac{0.80}{1.90} = 0.70 \quad \frac{0.60}{1.40} = 0.450 \]

\[ \Delta \lambda = (5.89)^2 \times 10^{-10} \quad \frac{0.450 + 0.377}{2 \times 2 \times 10^{-3} (19.60 - 7.77)} \]

\[ = 86.8 \times 10^{-8} \quad 0.827 \quad 11.83 \]

\[ = 6.06 \times 10^{-8} \text{ cm.} \]

\[ = 6.06 \text{ Angstrom Units with a probable error of} \]

\[ 0.16 \text{ Angstrom Units.} \]

**Discussion:** The sodium doublet spectral lines are a result of the electronic transitions $3^2S_{1/2} \rightarrow 3^2P_{1/2} (\lambda \ 5896)$ and $3^2S_{1/2} \rightarrow 3^2P_{3/2} (\lambda \ 5890)$. The wavelength difference is an indication of the difference in energy between the electronic states having the electron spin angular momentum aligned antiparallel ($\lambda \ 5896$) and parallel ($\lambda \ 5890$) with the electron orbital angular momentum around the nucleus.

\[ \lambda \nu = c \]

Where $\nu$ is frequency in cycles per second and $c$ is the velocity of light. Therefore:

\[ \Delta \nu = -\frac{c}{\lambda^2} \Delta \lambda \]

The energy difference between the two electronic levels is:

\[ \Delta E = h \Delta \nu = -\frac{hc}{\lambda^2} \Delta \lambda. \]

Where $h = 6.62 \times 10^{-27}$ erg - second is Planck's constant.
The accepted value of $\Delta \lambda$ for the sodium doublet is 5.96 Å which may be compared with the experiment quoted above. Using the value $\Delta \lambda = 5.96 \times 10^{-8}$ cm., the above formula for $\Delta E$ gives the result:

$$\Delta E = \frac{6.62 \times 10^{-27} \times 3 \times 10^{10} \times 5.96 \times 10^{-8}}{(5.89)^2 \times 10^{-10} \times 1.6 \times 10^{-12}}$$

$$= 2.13 \times 10^{-3} \text{ ev.}$$

**EXPERIMENT 2: Observation of Hyperfine Structure in the Mercury Green Line.**

**Procedure:** For this experiment a low pressure Mercury vapor lamp such as the Atomic Laboratories' Mercury Light Source should be used. A No. 74 Wratten filter or equivalent is placed between interferometer and diffuser plate in order to permit observation of the mercury green line, $\lambda 5461$, without interference from the other lines of the Mercury spectrum. Due to the faintness of the lines to be observed, one of the two diffuser plates on the Atomic Laboratories Mercury Light Source may be removed to enhance the intensity of the fringes. However, care should be taken to prevent direct exposure of the eye to the exposed lamp in the light source.

When good fringes have been obtained, place the telescope in a position such that the adjusting screws of the Fabry-Perot are accessible while the fringes are under observation through the telescope. Since fringes are much more conveniently obtained with a spacing of about 1 mm between the mirrors, and observation of the mercury hyperfine structure requires considerably increased spacing between the mirrors, this increase in spacing must be carefully accomplished in order not to lose the fringe pattern.

First loosen the carriage lock screw. Then, with a thumb on each side of the interferometer base, and index and middle fingers on each side of the carriage, very gently push the carriage away in order to increase the distance between the mirrors. (The carriage is moved by hand to achieve a sufficiently large separation.) Meanwhile carefully observe the fringe pattern for loss of sharpness of the fringes. When the fringes have become blurred, cease pushing and use the adjusting screws to restore sharpness to the fringes. Using this procedure, the spacing between the mirrors can be increased from one-half to one millimeter at a time.

As the spacing between the mirrors is gradually increased, a faint fringe will appear to move outward from each bright fringe and a yet fainter fringe will appear to move inward from each bright fringe. This effect will first become apparent when the spacing between the mirrors is about 5 millimeters. When the spacing between the mirrors is some 5 millimeters, each faint fringe will appear to have moved about one-fourth of the distance between bright fringes. When the spacing between the mirrors is some 10 millimeters, the faint fringes will appear to overlap about halfway between bright fringes. When the spacing between mirrors is some 20 millimeters, the faint fringes will appear to have become superimposed on the bright fringe once removed from the bright fringe from which they started.

At mirror spacings of about 5, 10 and 15 millimeters, make estimates of the location of the faint fringes with respect to the bright fringes. At these positions accurately measure the separation between the mirrors. This can be